

Theta Graph Designs

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Abstract

We solve the design spectrum problem for all theta graphs with 10, 11, 12, 13, 14 and 15 edges.

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1 Introduction

Let G be a simple graph. If the edge set of a simple graph K can be partitioned into edge sets of graphs each isomorphic to G , we say that there exists a *decomposition* of K into G . In the case where K is the complete graph K_n , we refer to the decomposition as a G *design* of order n . The *spectrum* of G is the set of non-negative integers n for which there exists a G design of order n . For completeness we remark that the empty set is a G design of order 0 as well as 1; these trivial cases will usually be omitted from discussion henceforth. A complete solution of the spectrum problem often seems to be very difficult. However it has been achieved in many cases, especially amongst the smaller graphs. We refer the reader to the survey article of Adams, Bryant and Buchanan, [2], and, for more up to date results, the Web site maintained by Bryant and McCourt, [5].

A *theta graph* $\Theta(a, b, c)$, $1 \leq a \leq b \leq c$, $b \geq 2$ is the graph with $a + b + c - 1$ vertices and $a + b + c$ edges that consists of three paths of lengths a , b and c with common end points but internally disjoint. Each of the two common end points of the paths has degree 3 and all other vertices have degree 2. Observe that $\Theta(a, b, c)$ is bipartite if $a \equiv b \equiv c \pmod{2}$, tripartite otherwise.

The aim of this paper is to establish the design spectrum for every theta graph with 10, 11, 12, 13, 14 or 15 edges. The design spectra for all theta graphs with 9 or fewer edges have been determined, [4]. Also there are some partial results for $\Theta(1, b, b)$, odd $b > 1$, [3], and it is known that there exists a $\Theta(a, b, c)$ design of order $2(a + b + c) + 1$, [10]. See [2, Section 5.5], specifically Theorems 5.9 and 5.10, for details and further references. For convenience, we record the main result of [10] as follows.

Table 1: Design existence conditions for theta graphs

$a + b + c$	conditions
10	$n \equiv 0, 1, 5, 16 \pmod{20}, n \neq 5$
12	$n \equiv 0, 1, 9, 16 \pmod{24}, n \neq 9$
14	$n \equiv 0, 1, 8, 21 \pmod{28}, n \neq 8$
15	$n \equiv 0, 1, 6, 10 \pmod{15}, n \neq 6, 10$
odd prime power ≥ 5	$n \equiv 0, 1 \pmod{a + b + c}$

Theorem 1.1 (Punnim & Pabhapote) *If $1 \leq a \leq b \leq c$ and $b \geq 2$, then there exists a $\Theta(a, b, c)$ design of order $2(a + b + c) + 1$.*

It is clear that a $\Theta(a, b, c)$ design of order n can exist only if (i) $n \leq 1$, or $n \geq a + b + c - 1$, and (ii) $n(n - 1) \equiv 0 \pmod{2(a + b + c)}$. These necessary conditions are determined by elementary counting and given explicitly in Table 1 for some values of $a + b + c$. In this paper we show that the conditions are sufficient for $n = 10, 11, 12, 13, 14$ and 15 . We state our results formally.

Theorem 1.2 *Designs of order n exist for all theta graphs $\Theta(a, b, c)$ with $a + b + c = 10$ if and only if $n \equiv 0, 1, 5$ or $16 \pmod{20}$ and $n \neq 5$.*

Theorem 1.3 *Designs of order n exist for all theta graphs $\Theta(a, b, c)$ with $a + b + c = 11$ if and only if $n \equiv 0$ or $1 \pmod{11}$.*

Theorem 1.4 *Designs of order n exist for all theta graphs $\Theta(a, b, c)$ with $a + b + c = 12$ if and only if $n \equiv 0, 1, 9$ or $16 \pmod{24}$ and $n \neq 9$.*

Theorem 1.5 *Designs of order n exist for all theta graphs $\Theta(a, b, c)$ with $a + b + c = 13$ if and only if $n \equiv 0$ or $1 \pmod{13}$.*

Theorem 1.6 *Designs of order n exist for all theta graphs $\Theta(a, b, c)$ with $a + b + c = 14$ if and only if $n \equiv 0, 1, 8$ or $21 \pmod{28}$ and $n \neq 8$.*

Theorem 1.7 *Designs of order n exist for all theta graphs $\Theta(a, b, c)$ with $a + b + c = 15$ if and only if $n \equiv 0, 1, 6$ or $10 \pmod{15}$ and $n \neq 6, 10$.*

2 Constructions

We use Wilson's fundamental construction involving group divisible designs, [12]. Recall that a K -GDD of type $g_1^{t_1} \dots g_r^{t_r}$ is an ordered triple $(V, \mathcal{G}, \mathcal{B})$ where V is a set of cardinality $v = t_1 g_1 + \dots + t_r g_r$, \mathcal{G} is a partition of V into t_i subsets each of cardinality g_i , $i = 1, \dots, r$, called *groups* and \mathcal{B} is a collection of subsets of cardinalities $k \in K$, called *blocks*, which collectively have the property that each pair of elements from different groups occurs in precisely one block but no pair of elements from the same group occurs at all. A $\{k\}$ -GDD is also called a k -GDD. As is well known, whenever q is a prime power there exists a q -GDD of type q^q and a $(q + 1)$ -GDD of type q^{q+1} (arising from affine and projective planes of order q respectively). A *parallel class* in a group divisible design is a subset of the block set in which each element of the base

set appears exactly once. A k -GDD is called *resolvable*, and denoted by k -RGDD, if the entire set of blocks can be partitioned into parallel classes.

Propositions 2.1 and 2.2 are for dealing with bipartite graphs, the latter being used only for 15-edge bipartite theta graphs, where a suitable decomposition of the form $K_{dr,ds}$ is not available. Propositions 2.3 to 2.8 deal with tripartite graphs.

Proposition 2.1 *Let d, f, r and s be positive integers, let g be a non-negative integer, and let $e = 0$ or 1 . Suppose there exist G designs of orders $fdrs + e$ and $gds + e$, and suppose there exists a decomposition into G of the complete bipartite graph $K_{dr,ds}$. Then there exist G designs of order $tfdrs + gds + e$ for $t = 0, 1, 2, \dots$*

Proof If $t = 0$, there is nothing to prove; so we may assume $t \geq 1$. Let $g = 0$ and use induction on t . Assume there exists a G design of order $tfdrs + e$ for some $t \geq 1$. Take a complete bipartite graph $K_{tf,fs}$ and inflate the first part by a factor dr and the second part by a factor of ds , so that the edges become $K_{dr,ds}$ graphs. If $e = 1$, add an extra point, ∞ . Overlay the inflated parts, together with ∞ when $e = 1$, with $K_{tfdrs+e}$ or K_{fdrs+e} as appropriate. The result is a complete graph $K_{(t+1)fdrs+e}$ which admits a decomposition into G since there exist decompositions into G of the components, $K_{tfdrs+e}$, K_{fdrs+e} and $K_{dr,ds}$.

For $g \geq 1$, the same construction but starting with a complete bipartite graph $K_{tf,g}$ yields a design of order $K_{tfdrs+gds+e}$ since we may now assume that there exists a design of order $tfdrs + e$. \square

Proposition 2.2 *Let r and s be not necessarily distinct positive integers, and let $e = 0$ or 1 . Suppose there exist G designs of orders $r + e$ and (if $s \neq r$) $s + e$, and suppose there exist decompositions into G of the complete bipartite graphs $K_{r,r}$ and (if $s \neq r$) $K_{r,s}$. Then there exist G designs of order $tr + s + e$ for $t = 0, 1, 2, \dots$*

Proof If $t = 0$, there is nothing to prove; so we may assume $t \geq 1$. Start with the graph K_{t+1} , inflate one point by a factor of s and all others by r so that the original edges become $K_{r,s}$ graphs and, when $t \geq 2$, $K_{r,r}$ graphs. Add a new point, ∞ , if $e = 1$. Overlay the r -inflated parts together with ∞ if $e = 1$ by K_{r+e} . Overlay the s -inflated part together with ∞ if $e = 1$ by K_{s+e} . The result is a graph K_{tr+s+e} which admits a decomposition into G . \square

Proposition 2.3 *Let p be a positive integer. Suppose there exist G designs of orders $p, p + 1, 2p$ and $2p + 1$. Suppose also there exist decompositions into G of $K_{p,p,p}$, $K_{p,p,p,p}$ and $K_{p,p,p,p,p}$. Then there exist G designs of order n for $n \equiv 0, 1 \pmod{p}$.*

Proof It is known that there exists a $\{3, 4, 5\}$ -GDD of type 1^t for $t \geq 3$, $t \neq 6, 8$; see [1]. Inflate each point of the GDD by a factor of p , thus expanding the blocks to complete multipartite graphs $K_{p,p,p}$, $K_{p,p,p,p}$ and $K_{p,p,p,p,p}$. Let $e = 0$ or 1 . If $e = 1$, add an extra point, ∞ . Overlay each inflated group, together with ∞ if $e = 1$, with K_{p+e} . Since a design of order $p + e$ and decompositions into G of $K_{p,p,p}$, $K_{p,p,p,p}$ and $K_{p,p,p,p,p}$ exist, this construction creates a design of order $pt + e$ for $t \geq 3$, $t \neq 6, 8$.

For order $6p + e$, use a p -inflated 3-GDD of type 2^3 , plus an extra point if $e = 1$, with decompositions into G of K_{2p+e} and $K_{p,p,p}$. Similarly, for order $8p + e$, use a 3-GDD of type 2^4 instead. \square

Proposition 2.4 *Let p be a positive integer, let $f = 0$ or 1 and write f' for $1 - f$. Suppose there exist G designs of orders $3p + f'$, $4p$, $4p + 1$, $5p + f$, $7p + f'$, $8p$, $8p + 1$, $9p + f$, $11p + f'$, $13p + f$ and*

Table 2: The construction for Proposition 2.4

$12pt + 4p,$	$x = 0, y = 0, z = 0,$	$e = 0,$	missing $4p$
$12pt + 4p + 4p,$	$x = 0, y = 0, z = 1,$	$e = 0,$	missing $8p$
$12pt + 4p + 8p,$	$x = 0, y = 0, z = 2,$	$e = 0,$	missing $12p$
$12pt + 4p + 1,$	$x = 0, y = 0, z = 0,$	$e = 1,$	missing $4p + 1$
$12pt + 4p + 4p + 1,$	$x = 0, y = 0, z = 1,$	$e = 1,$	missing $8p + 1$
$12pt + 4p + 8p + 1,$	$x = 0, y = 0, z = 2,$	$e = 1,$	missing $12p + 1$
$12pt + 4p + 5p + f,$	$x = 1, y = 0, z = 1,$	$e = f,$	missing $9p + f$
$12pt + 4p + 9p + f,$	$x = 1, y = 0, z = 2,$	$e = f,$	missing $13p + f$
$12pt + 4p + 13p + f,$	$x = 1, y = 0, z = 3,$	$e = f,$	missing $5p + f, 17p + f$
$12pt + 4p + 3p + f',$	$x = 0, y = 1, z = 0,$	$e = f',$	missing $7p + f'$
$12pt + 4p + 7p + f',$	$x = 0, y = 1, z = 1,$	$e = f',$	missing $11p + f'$
$12pt + 4p + 11p + f',$	$x = 0, y = 1, z = 2,$	$e = f',$	missing $3p + f', 15p + f'$

suppose there exist decompositions into G of $K_{p,p,p,p}$, $K_{p,p,p,p,p}$, $K_{p,p,p,p,3p}$ and $K_{p,p,p,p,4p}$, $K_{2p,2p,2p}$ and $K_{4p,4p,4p,5p}$. Then there exist G designs of order n for $n \equiv 0, 1, p + f, 3p + f' \pmod{4p}$, $n \neq p + f$.

Proof Start with a 4-RGDD of type 4^{3t+1} , $t \geq 1$, [8]; see also [9]. There are $4t$ parallel classes. Let $x, y, z \geq 0$ and $w = x + y + z \leq 4$. If $w > 0$, add a new group of size w and adjoin each point of this new group to all blocks of a parallel class to create a $\{4, 5\}$ -GDD of type $4^{3t+1}w^1$, $t \geq 1$, which degenerates to a 5-GDD of type 4^5 if $t = 1$ and $w = 4$. In the new group inflate x points by a factor of p , y points by a factor of $3p$ and z points by a factor of $4p$. Inflate all points in the other groups by a factor of p . So the original blocks become $K_{p,p,p,p}$ graphs, and new ones $K_{p,p,p,p,p}$ or $K_{p,p,p,p,3p}$ or $K_{p,p,p,p,4p}$ graphs. Either overlay the groups with K_{4p} and, if $w > 0$, $K_{px+3py+4pz}$, or add a new point and overlay with K_{4p+1} and, if $w > 0$, $K_{px+3py+4pz+1}$. Using decompositions into G of K_{4p} , K_{4p+1} , $K_{p,p,p,p}$, $K_{p,p,p,p,p}$, $K_{p,p,p,p,3p}$ and $K_{p,p,p,p,4p}$ this construction yields a design of order $12pt + 4p + px + 3py + 4pz + e$ if $t \geq 1$, where $e = 0$ or 1 , whenever a design of order $px + 3py + 4pz + e$ exists.

As illustrated in Table 2, all design orders $n \equiv 0, 1, p + f, 3p + f' \pmod{4p}$, $n \neq p + f$, are covered except for those values indicated as missing. The missing values not assumed as given are handled independently as follows. See [7, Tables 4.3 and 4.10] for lists of small 3- and 4-GDDs.

For $12p + e$, use decompositions of K_{4p+e} and $K_{2p,2p,2p}$ with a $2p$ -inflated 3-GDD of type 2^3 plus an extra point if $e = 1$.

For $15p + f'$, use decompositions of $K_{3p+f'}$ and $K_{p,p,p,p}$ with a p -inflated 4-GDD of type 3^5 plus an extra point if $f' = 1$.

For $17p + f$, use decompositions of K_{4p+f} , K_{5p+f} , $K_{4p,4p,4p,5p}$ with the trivial 4-GDD of type 1^4 plus an extra point if $f = 1$. \square

Proposition 2.5 Suppose there exist G designs of orders 16, 20, 21, 25, 36, 40, 41, 45, 56, 65 and suppose there exist decompositions into G of $K_{10,10,10}$, $K_{5,5,5,5}$, $K_{20,20,20,25}$, $K_{5,5,5,5,5}$, $K_{5,5,5,5,15}$ and $K_{5,5,5,5,20}$. Then there exist G designs of order n for $n \equiv 0, 1, 5, 16 \pmod{20}$, $n \neq 5$.

Proof Use Proposition 2.4 with $p = 5$ and $f = 0$. \square

Table 3: The construction for Proposition 2.6

$48t + 24,$	$x = 0, y = 0,$	$e = 0,$	missing 24
$48t + 24 + 24,$	$x = 0, y = 1,$	$e = 0,$	missing 48
$48t + 24 + 1,$	$x = 0, y = 0,$	$e = 1,$	missing 25
$48t + 24 + 25,$	$x = 0, y = 1,$	$e = 1,$	missing 49
$48t + 24 + 33,$	$x = 1, y = 1,$	$e = 1,$	missing 57
$48t + 24 + 57,$	$x = 1, y = 2,$	$e = 1,$	missing 33, 81
$48t + 24 + 16,$	$x = 2, y = 0,$	$e = 0,$	missing 40
$48t + 24 + 40,$	$x = 2, y = 1,$	$e = 0,$	missing 16, 64

Proposition 2.6 *Suppose there exist G designs of orders 16, 24, 25, 33, 40, 49, 57, 81 and suppose there exist decompositions into G of $K_{8,8,8}$, $K_{8,8,8,8}$ and $K_{8,8,8,24}$. Then there exist G designs of order n for $n \equiv 0, 1, 9, 16 \pmod{24}$, $n \neq 9$.*

Proof Start with a 3-RGDD of type 3^{2t+1} , $t \geq 1$, [11] (see also [9]), which has $3t$ parallel classes. Let $x, y \geq 0$, $x + y \leq 3$ and if $x + y > 0$, add a new group of size $x + y$ and adjoin each point of this new group to all blocks of a parallel class to create a $\{3,4\}$ -GDD of type $3^{2t+1}(x+y)^1$. This degenerates to a 4-GDD of type 3^4 if $t = 1$ and $x + y = 3$. In the new group inflate x points by a factor of 8 and y points by a factor of 24. Inflate all other points by a factor of 8. The original blocks become $K_{8,8,8}$ graphs and new ones $K_{8,8,8,8}$ or $K_{8,8,8,24}$ graphs. Either overlay the groups with K_{24} and possibly K_{8x+24y} , or add a new point and overlay the groups with K_{25} and possibly $K_{8x+24y+1}$. Using decompositions into G of K_{24} , K_{25} , $K_{8,8,8}$, $K_{8,8,8,8}$ and $K_{8,8,8,24}$ this construction yields a design of order $48t + 24 + 8x + 24y + e$, $t \geq 1$, $e = 0$ or 1 , whenever a design of order $8x + 24y + e$ exists.

As illustrated in Table 3, all design orders $n \equiv 0, 1, 9, 16 \pmod{24}$, $n \neq 9$, are covered except for those values indicated as missing. The missing values not assumed as given in the statement of the proposition, namely 48 and 64, are constructed using decompositions of K_{16} and $K_{8,8,8}$ with an 8-inflated 3-GDD of type 2^3 for 48, or type 2^4 for 64. \square

Proposition 2.7 *Suppose there exist G designs of orders 21, 28, 29, 36, 49, 56, 57, 64, 77, 92 and suppose there exist decompositions into G of $K_{14,14,14}$, $K_{7,7,7,7}$, $K_{28,28,28,35}$, $K_{7,7,7,7,7}$, $K_{7,7,7,7,21}$ and $K_{7,7,7,7,28}$. Then there exist G designs of order n for $n \equiv 0, 1, 8, 21 \pmod{28}$, $n \neq 8$.*

Proof Use Proposition 2.4 with $p = 7$ and $f = 1$. \square

Proposition 2.8 *Suppose there exist G designs of orders 15, 16, 21, 25, 30, 31, 36, 40, 51, 55, 66 and 70, and suppose there exist decompositions into G of $K_{5,5,5}$ and $K_{5,5,5,5}$. Then there exist G designs of order n for $n \equiv 0, 1, 6, 10 \pmod{15}$, $n \neq 6, 10$.*

Proof Start with a 3-RGDD of type 3^{2t+1} , $t \geq 1$, [11] (see also [9]), which has $3t$ parallel classes. Let $w \geq 0$, $w \leq 3t$, and if $w > 0$, add a new group of size w and adjoin each point of this new group to all blocks of a parallel class to create a $\{3,4\}$ -GDD of type $3^{2t+1}w^1$. This degenerates to a 4-GDD of type 3^4 if $t = 1$ and $w = 3$. Inflate all points by a factor of 5, so that the original blocks become $K_{5,5,5}$ graphs and new ones $K_{5,5,5,5}$ graphs. Let $e = 0$ or 1 , and if $e = 1$, add a new point, ∞ . Overlay each group, including ∞ if $e = 1$, with K_{15+e} or K_{5w+e} ,

Table 4: The construction for Proposition 2.8

$30t + 15,$	$w = 0, e = 0,$	$t \geq 1,$	missing 15
$30t + 15 + 15,$	$w = 3, e = 0,$	$t \geq 1,$	missing 30
$30t + 15 + 1,$	$w = 0, e = 1,$	$t \geq 1,$	missing 16
$30t + 15 + 16,$	$w = 3, e = 1,$	$t \geq 1,$	missing 31
$30t + 15 + 21,$	$w = 4, e = 1,$	$t \geq 2,$	missing 36, 66
$30t + 15 + 36,$	$w = 7, e = 1,$	$t \geq 3,$	missing 21, 51, 81, 111
$30t + 15 + 25,$	$w = 5, e = 0,$	$t \geq 2,$	missing 40, 70
$30t + 15 + 40,$	$w = 8, e = 0,$	$t \geq 3,$	missing 25, 55, 85, 115

as appropriate. Using decompositions of K_{15+e} , $K_{5,5,5}$, and $K_{5,5,5,5}$ this construction yields a design of order $30t + 15 + 5w + e$, $t \geq w/3$, $e = 0$ or 1 , whenever a design of order $5w + e$ exists.

As illustrated in Table 4, all design orders $n \equiv 0, 1, 6, 10 \pmod{15}$, $n \neq 6, 10$, are covered except for those values indicated as missing. The missing values not assumed as given are constructed as follows. See [7, Table 4.3] for a list of small 3-GDDs.

For 81, use decompositons of K_{21} and $K_{5,5,5}$ with a 5-inflated 3-GDD of type 4^4 and an extra point.

For 85, use decompositons of K_{15} , K_{25} and $K_{5,5,5}$ with a 5-inflated 3-GDD of type $3^4 5^1$.

For 111, use decompositons of K_{21} , K_{31} and $K_{5,5,5}$ with a 5-inflated 3-GDD of type $4^4 6^1$ and an extra point.

For 115, use decompositons of K_{15} , K_{25} and $K_{5,5,5}$ with a 5-inflated 3-GDD of type $3^1 5^4$. \square

3 Proofs of the theorems

We deal with each of the theta graphs stated in the theorems. The lemmas in this section assert the existence of specific graph designs and decompositions of multipartite graphs. These are used as ingredients for the propositions of Section 2 to construct the decompositions of complete graphs required to prove the theorems. With a few exceptions, the details of the decompositions that constitute the proofs of the lemmas have been deferred to sections in the rather lengthy Appendix to this paper. If absent, the Appendix may be obtained from the ArXiv, or by request from the first author. In the presentation of our results we represent $\Theta(a, b, c)$ by a subscripted ordered $(a + b + c - 1)$ -tuple $(v_1, v_2, \dots, v_{a+b+c-1})_{\Theta(a,b,c)}$, where v_1 and v_2 are the vertices of degree 3 and the three paths are

- (i) (v_1, v_2) if $a = 1$, $(v_1, v_3, \dots, v_{a+1}, v_2)$ if $a \geq 2$,
- (ii) $(v_1, v_{a+2}, \dots, v_{a+b}, v_2)$,
- (iii) $(v_1, v_{a+b+1}, \dots, v_{a+b+c-1}, v_2)$.

If a graph G has e edges, the numbers of occurrences of G in a decomposition into G of the complete graph K_n , the complete r -partite graph K_{n^r} and the complete $(r + 1)$ -partite graph $K_{n^r m^1}$ are respectively

$$\frac{n(n-1)}{2e}, \quad \frac{n^2 r(r-1)}{2e} \quad \text{and} \quad \frac{nr(n(r-1) + 2m)}{2e}.$$

The number of theta graphs with e edges is $\lfloor e^2/12 - 1/2 \rfloor$ of which $\lfloor e^2/48 + (e \bmod 2)(e - 8)/8 + 1/2 \rfloor$ are bipartite; see [6] for example.

All decompositions were created by a computer program written in the C language, and a MATHEMATICA program provided final assurance regarding their correctness. Most of the decompositions were obtained without difficulty. However, there were a few that seemed to present a challenge, notably K_{20} for theta graphs of 10 edges, K_{24} for theta graphs of 12 edges and K_{30} for theta graphs of 15 edges. So, for these three cases, it is appropriate to include the decomposition details in the main body of the paper.

Lemma 3.1 *There exist $\Theta(a, b, c)$ designs of orders 16, 20 and 25 for $a + b + c = 10$. There exist $\Theta(a, b, c)$ designs of orders 36, 40, 41, 45, 56, 65 for $a + b + c = 10$ with a, b, c not all even.*

Proof There are seven theta graphs of which two are bipartite. We present designs of order 20 here; the remaining decompositions are presented in Appendix A.

K_{20} Let the vertex set be Z_{20} . The decompositions consist of the graphs

$(4, 17, 0, 9, 7, 1, 12, 15, 13)_{\Theta(1,2,7)}, (11, 17, 18, 10, 1, 13, 3, 7, 16)_{\Theta(1,2,7)},$
 $(1, 6, 18, 14, 11, 12, 2, 0, 15)_{\Theta(1,2,7)},$
 $(12, 4, 10, 18, 0, 8, 15, 3, 11)_{\Theta(1,2,7)}, (6, 2, 8, 11, 19, 7, 0, 14, 16)_{\Theta(1,2,7)},$
 $(2, 4, 18, 7, 15, 10, 14, 8, 16)_{\Theta(1,2,7)}, (6, 12, 0, 10, 16, 3, 18, 14, 19)_{\Theta(1,2,7)},$
 $(0, 1, 9, 18, 12, 15, 4, 3, 5)_{\Theta(1,3,6)}, (0, 7, 17, 5, 10, 14, 12, 8, 2)_{\Theta(1,3,6)},$
 $(8, 6, 14, 1, 13, 3, 17, 4, 19)_{\Theta(1,3,6)},$
 $(3, 15, 2, 1, 11, 17, 18, 7, 6)_{\Theta(1,3,6)}, (3, 7, 14, 11, 19, 5, 6, 18, 10)_{\Theta(1,3,6)},$
 $(7, 19, 15, 18, 13, 14, 2, 10, 11)_{\Theta(1,3,6)}, (15, 19, 11, 2, 14, 6, 3, 9, 10)_{\Theta(1,3,6)},$
 $(0, 2, 3, 6, 9, 11, 10, 14, 8)_{\Theta(1,4,5)}, (17, 2, 7, 12, 13, 9, 0, 18, 11)_{\Theta(1,4,5)},$
 $(6, 11, 5, 3, 17, 18, 8, 0, 7)_{\Theta(1,4,5)},$
 $(8, 12, 13, 15, 16, 4, 3, 11, 9)_{\Theta(1,4,5)}, (1, 8, 3, 15, 7, 16, 13, 0, 5)_{\Theta(1,4,5)},$
 $(1, 17, 5, 7, 19, 4, 9, 16, 0)_{\Theta(1,4,5)}, (12, 17, 5, 9, 13, 11, 19, 0, 4)_{\Theta(1,4,5)},$
 $(2, 1, 8, 10, 18, 7, 12, 0, 6)_{\Theta(2,2,6)}, (10, 15, 7, 8, 12, 11, 6, 13, 5)_{\Theta(2,2,6)},$
 $(10, 8, 11, 9, 0, 16, 7, 14, 17)_{\Theta(2,2,6)},$
 $(5, 1, 7, 3, 8, 13, 11, 17, 15)_{\Theta(2,2,6)}, (1, 9, 4, 5, 19, 3, 7, 13, 15)_{\Theta(2,2,6)},$
 $(5, 17, 0, 19, 11, 9, 13, 16, 1)_{\Theta(2,2,6)}, (9, 17, 3, 12, 7, 11, 15, 19, 13)_{\Theta(2,2,6)},$
 $(0, 5, 18, 2, 10, 4, 9, 8, 17)_{\Theta(2,3,5)}, (13, 5, 2, 11, 9, 3, 7, 6, 12)_{\Theta(2,3,5)},$
 $(7, 6, 0, 1, 15, 2, 12, 9, 10)_{\Theta(2,3,5)},$
 $(11, 3, 12, 16, 15, 0, 8, 7, 4)_{\Theta(2,3,5)}, (3, 7, 10, 0, 15, 8, 16, 4, 19)_{\Theta(2,3,5)},$
 $(7, 19, 16, 14, 11, 12, 4, 15, 2)_{\Theta(2,3,5)}, (11, 19, 8, 3, 6, 18, 15, 12, 0)_{\Theta(2,3,5)},$
 $(8, 2, 7, 5, 13, 9, 4, 18, 17)_{\Theta(2,4,4)}, (18, 6, 10, 9, 8, 0, 7, 17, 4)_{\Theta(2,4,4)},$
 $(13, 11, 15, 8, 18, 0, 10, 3, 5)_{\Theta(2,4,4)},$
 $(7, 3, 6, 13, 4, 11, 15, 0, 9)_{\Theta(2,4,4)}, (3, 7, 0, 2, 19, 4, 8, 11, 10)_{\Theta(2,4,4)},$
 $(15, 19, 12, 3, 16, 5, 8, 17, 11)_{\Theta(2,4,4)}, (15, 19, 18, 1, 12, 7, 14, 11, 16)_{\Theta(2,4,4)},$
 $(15, 10, 18, 13, 6, 12, 11, 0, 8)_{\Theta(3,3,4)}, (0, 11, 16, 3, 17, 8, 10, 1, 5)_{\Theta(3,3,4)},$
 $(5, 7, 0, 13, 3, 8, 17, 19, 14)_{\Theta(3,3,4)},$
 $(6, 18, 7, 17, 13, 12, 0, 1, 2)_{\Theta(3,3,4)}, (6, 10, 5, 4, 14, 18, 2, 16, 17)_{\Theta(3,3,4)},$
 $(10, 14, 9, 8, 11, 1, 2, 3, 13)_{\Theta(3,3,4)}, (14, 18, 10, 6, 15, 5, 2, 9, 19)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 4 \pmod{20}$ for only the first three graphs in each design. □

Lemma 3.2 *There exist decompositions into $\Theta(a, b, c)$ of the complete bipartite graph $K_{10,5}$ for $a + b + c = 10$ with a, b, c all even. There exist decompositions into $\Theta(a, b, c)$ of the complete multipartite graphs $K_{10,10,10}$, $K_{5,5,5,5}$, $K_{20,20,20,25}$, $K_{5,5,5,5,5}$, $K_{5,5,5,5,15}$ and $K_{5,5,5,5,20}$ for $a + b + c = 10$ with a, b, c not all even.*

Proof The decompositions are presented in Appendix A. □

Theorem 1.2 for $\Theta(a, b, c)$ with $a + b + c = 10$ follows from Theorem 1.1, Lemmas 3.1 and 3.2, and Proposition 2.1 (with $d = 5$, $r = 2$, $s = 1$, $f = 2$ and $g = 0, 3$ or 5) if a, b and c are all even, Proposition 2.5 otherwise. □

Lemma 3.3 *There exist $\Theta(a, b, c)$ designs of orders 11 and 12 for $a + b + c = 11$. There exists a $\Theta(a, b, c)$ design of order 22 for $a + b + c = 11$ with a, b, c not all odd.*

Proof There are nine theta graphs of which three are bipartite. The decompositions are presented in Appendix B. □

Lemma 3.4 *There exist decompositions into $\Theta(a, b, c)$ of the complete bipartite graph $K_{11,11}$ for $a + b + c = 11$ with a, b, c all odd. There exist decompositions into $\Theta(a, b, c)$ of the complete multipartite graphs $K_{11,11,11}$, $K_{11,11,11,11}$ and $K_{11,11,11,11,11}$ for $a + b + c = 11$ with a, b, c not all odd.*

Proof The decompositions are presented in Appendix B. □

Theorem 1.3 for $\Theta(a, b, c)$ with $a + b + c = 11$ follows from Theorem 1.1, Lemmas 3.3 and 3.4, and Proposition 2.1 (with $d = 11$, $r = s = f = 1$ and $g = 0$) if a, b and c are all odd, Proposition 2.3 otherwise. □

Lemma 3.5 *There exist $\Theta(a, b, c)$ designs of orders 16, 24 and 33 for $a + b + c = 12$. There exist $\Theta(a, b, c)$ designs of orders 40, 49, 57 and 81 for $a + b + c = 12$ with a, b, c not all even.*

Proof There are eleven theta graphs of which three are bipartite. We present designs of order 24 here; the remaining decompositions are presented in Appendix C.

K_{24} Let the vertex set be Z_{24} . The decompositions consist of

- $(11, 9, 16, 19, 10, 12, 6, 15, 7, 2, 8)_{\Theta(1,2,9)}$, $(8, 0, 10, 7, 6, 19, 18, 1, 20, 22, 12)_{\Theta(1,2,9)}$,
- $(2, 6, 13, 15, 1, 17, 18, 14, 22, 19, 21)_{\Theta(1,2,9)}$, $(10, 7, 1, 3, 21, 8, 14, 9, 19, 16, 13)_{\Theta(1,2,9)}$,
- $(16, 21, 1, 7, 9, 22, 15, 19, 5, 17, 4)_{\Theta(1,2,9)}$, $(18, 4, 10, 21, 17, 14, 16, 6, 11, 12, 19)_{\Theta(1,2,9)}$,
- $(21, 20, 7, 2, 14, 13, 18, 6, 5, 4, 12)_{\Theta(1,2,9)}$, $(21, 12, 13, 22, 10, 5, 15, 20, 16, 3, 9)_{\Theta(1,2,9)}$,
- $(4, 20, 0, 1, 19, 23, 11, 15, 3, 7, 5)_{\Theta(1,2,9)}$, $(4, 23, 13, 8, 15, 12, 7, 0, 11, 17, 20)_{\Theta(1,2,9)}$,
- $(12, 23, 16, 8, 19, 7, 4, 15, 13, 5, 21)_{\Theta(1,2,9)}$,
- $(22, 18, 1, 11, 8, 16, 17, 13, 9, 15, 2)_{\Theta(1,3,8)}$, $(9, 20, 10, 22, 1, 3, 21, 4, 19, 12, 2)_{\Theta(1,3,8)}$,
- $(7, 4, 17, 5, 10, 12, 21, 6, 19, 16, 20)_{\Theta(1,3,8)}$, $(16, 2, 3, 9, 21, 22, 0, 18, 13, 15, 11)_{\Theta(1,3,8)}$,
- $(18, 19, 14, 7, 9, 12, 11, 13, 0, 6, 22)_{\Theta(1,3,8)}$, $(12, 7, 16, 11, 17, 22, 4, 15, 10, 13, 6)_{\Theta(1,3,8)}$,
- $(8, 15, 17, 0, 10, 21, 11, 19, 5, 18, 16)_{\Theta(1,3,8)}$, $(5, 21, 13, 7, 15, 6, 11, 3, 22, 9, 0)_{\Theta(1,3,8)}$,
- $(7, 23, 8, 5, 22, 12, 0, 2, 13, 16, 1)_{\Theta(1,3,8)}$, $(8, 23, 1, 14, 20, 6, 17, 15, 7, 9, 16)_{\Theta(1,3,8)}$,
- $(13, 23, 21, 15, 3, 19, 14, 4, 16, 7, 0)_{\Theta(1,3,8)}$,
- $(22, 19, 10, 9, 20, 11, 16, 8, 2, 21, 4)_{\Theta(1,4,7)}$, $(19, 9, 3, 21, 18, 14, 1, 7, 2, 20, 4)_{\Theta(1,4,7)}$,

$(20, 18, 15, 0, 3, 6, 2, 16, 21, 10, 11)_{\Theta(1,4,7)}, (21, 0, 7, 20, 22, 14, 18, 16, 15, 3, 13)_{\Theta(1,4,7)},$
 $(4, 11, 7, 18, 15, 8, 12, 0, 9, 17, 13)_{\Theta(1,4,7)}, (4, 1, 22, 13, 15, 18, 2, 9, 11, 7, 0)_{\Theta(1,4,7)},$
 $(6, 22, 1, 8, 14, 0, 11, 17, 5, 20, 21)_{\Theta(1,4,7)}, (7, 9, 15, 14, 6, 22, 8, 19, 1, 13, 21)_{\Theta(1,4,7)},$
 $(1, 22, 5, 21, 15, 23, 7, 14, 13, 9, 16)_{\Theta(1,4,7)}, (6, 23, 7, 13, 5, 16, 3, 9, 14, 17, 22)_{\Theta(1,4,7)},$
 $(15, 17, 23, 14, 0, 6, 5, 4, 13, 12, 21)_{\Theta(1,4,7)},$
 $(1, 6, 22, 19, 9, 8, 5, 13, 12, 11, 17)_{\Theta(1,5,6)}, (0, 7, 9, 17, 19, 4, 8, 3, 16, 2, 21)_{\Theta(1,5,6)},$
 $(5, 17, 19, 16, 9, 15, 6, 4, 22, 12, 18)_{\Theta(1,5,6)}, (3, 15, 22, 16, 12, 10, 21, 8, 20, 9, 4)_{\Theta(1,5,6)},$
 $(13, 4, 22, 15, 7, 12, 16, 20, 3, 5, 18)_{\Theta(1,5,6)}, (15, 1, 16, 6, 22, 10, 0, 2, 19, 18, 21)_{\Theta(1,5,6)},$
 $(18, 2, 10, 19, 3, 11, 1, 4, 21, 14, 15)_{\Theta(1,5,6)}, (5, 7, 0, 18, 15, 11, 12, 9, 2, 6, 10)_{\Theta(1,5,6)},$
 $(7, 13, 22, 11, 19, 6, 3, 14, 23, 2, 8)_{\Theta(1,5,6)}, (10, 23, 14, 18, 22, 5, 17, 20, 13, 15, 19)_{\Theta(1,5,6)},$
 $(21, 23, 16, 10, 2, 22, 15, 6, 7, 18, 3)_{\Theta(1,5,6)},$
 $(11, 15, 2, 13, 19, 1, 0, 4, 12, 3, 20)_{\Theta(2,2,8)}, (16, 13, 18, 1, 21, 17, 4, 14, 15, 0, 10)_{\Theta(2,2,8)},$
 $(12, 11, 18, 0, 14, 19, 20, 5, 21, 7, 15)_{\Theta(2,2,8)}, (13, 3, 17, 14, 20, 18, 15, 19, 22, 0, 8)_{\Theta(2,2,8)},$
 $(12, 14, 2, 17, 13, 19, 18, 9, 22, 7, 8)_{\Theta(2,2,8)}, (13, 20, 7, 0, 3, 1, 15, 8, 5, 22, 17)_{\Theta(2,2,8)},$
 $(2, 17, 9, 18, 22, 4, 7, 19, 16, 6, 10)_{\Theta(2,2,8)}, (14, 2, 6, 10, 18, 22, 15, 3, 0, 17, 1)_{\Theta(2,2,8)},$
 $(1, 2, 7, 8, 23, 17, 15, 12, 6, 22, 13)_{\Theta(2,2,8)}, (9, 10, 15, 16, 7, 14, 22, 8, 11, 23, 18)_{\Theta(2,2,8)},$
 $(14, 18, 0, 5, 20, 23, 6, 21, 10, 9, 1)_{\Theta(2,2,8)},$
 $(7, 11, 0, 1, 14, 3, 15, 10, 22, 17, 16)_{\Theta(2,3,7)}, (1, 20, 15, 17, 3, 8, 18, 5, 22, 14, 4)_{\Theta(2,3,7)},$
 $(7, 0, 13, 6, 8, 16, 4, 9, 2, 12, 14)_{\Theta(2,3,7)}, (10, 14, 13, 1, 5, 2, 3, 21, 19, 17, 18)_{\Theta(2,3,7)},$
 $(9, 7, 21, 20, 18, 6, 19, 15, 22, 4, 5)_{\Theta(2,3,7)}, (1, 3, 19, 4, 10, 16, 13, 21, 2, 6, 0)_{\Theta(2,3,7)},$
 $(3, 11, 12, 13, 20, 18, 16, 15, 4, 8, 2)_{\Theta(2,3,7)}, (13, 15, 17, 4, 7, 8, 10, 19, 20, 0, 18)_{\Theta(2,3,7)},$
 $(5, 23, 12, 0, 2, 9, 7, 10, 16, 21, 1)_{\Theta(2,3,7)}, (15, 21, 12, 6, 11, 23, 20, 7, 22, 3, 4)_{\Theta(2,3,7)},$
 $(19, 23, 14, 4, 0, 5, 20, 16, 12, 8, 7)_{\Theta(2,3,7)},$
 $(4, 5, 18, 0, 12, 3, 7, 16, 6, 8, 15)_{\Theta(2,4,6)}, (1, 16, 13, 10, 6, 22, 5, 11, 2, 19, 8)_{\Theta(2,4,6)},$
 $(22, 21, 4, 12, 11, 16, 5, 14, 3, 17, 15)_{\Theta(2,4,6)}, (18, 9, 17, 8, 4, 5, 0, 13, 21, 12, 14)_{\Theta(2,4,6)},$
 $(15, 17, 10, 19, 12, 7, 4, 1, 20, 18, 6)_{\Theta(2,4,6)}, (3, 5, 19, 15, 9, 6, 1, 12, 4, 10, 7)_{\Theta(2,4,6)},$
 $(10, 3, 2, 8, 17, 0, 11, 6, 15, 22, 7)_{\Theta(2,4,6)}, (3, 18, 22, 9, 0, 2, 6, 7, 15, 14, 19)_{\Theta(2,4,6)},$
 $(10, 18, 13, 5, 2, 21, 23, 0, 1, 8, 7)_{\Theta(2,4,6)}, (16, 23, 15, 1, 19, 22, 9, 8, 11, 14, 7)_{\Theta(2,4,6)},$
 $(16, 23, 19, 18, 10, 14, 17, 11, 15, 2, 6)_{\Theta(2,4,6)},$
 $(15, 14, 21, 3, 0, 4, 11, 19, 20, 1, 13)_{\Theta(2,5,5)}, (14, 0, 18, 15, 2, 4, 19, 8, 16, 1, 5)_{\Theta(2,5,5)},$
 $(18, 1, 11, 9, 8, 6, 21, 19, 17, 0, 15)_{\Theta(2,5,5)}, (10, 21, 2, 13, 5, 19, 8, 7, 14, 0, 20)_{\Theta(2,5,5)},$
 $(21, 13, 4, 3, 11, 17, 15, 0, 7, 1, 2)_{\Theta(2,5,5)}, (19, 22, 10, 21, 7, 15, 11, 14, 12, 18, 1)_{\Theta(2,5,5)},$
 $(2, 7, 12, 0, 10, 20, 8, 6, 17, 1, 4)_{\Theta(2,5,5)}, (10, 18, 8, 15, 6, 12, 4, 14, 9, 20, 23)_{\Theta(2,5,5)},$
 $(4, 22, 17, 14, 1, 12, 9, 20, 15, 16, 18)_{\Theta(2,5,5)}, (7, 12, 22, 2, 16, 4, 15, 20, 6, 14, 23)_{\Theta(2,5,5)},$
 $(20, 22, 14, 12, 0, 23, 4, 17, 9, 1, 6)_{\Theta(2,5,5)},$
 $(14, 13, 20, 7, 16, 9, 22, 17, 18, 21, 0)_{\Theta(3,3,6)}, (21, 9, 3, 5, 22, 18, 2, 16, 1, 17, 19)_{\Theta(3,3,6)},$
 $(21, 12, 4, 13, 16, 18, 14, 15, 20, 17, 11)_{\Theta(3,3,6)}, (19, 2, 22, 8, 15, 9, 12, 1, 13, 21, 11)_{\Theta(3,3,6)},$
 $(7, 2, 21, 10, 9, 4, 14, 17, 16, 22, 12)_{\Theta(3,3,6)}, (18, 14, 7, 2, 15, 1, 19, 4, 16, 8, 23)_{\Theta(3,3,6)},$
 $(8, 19, 7, 0, 11, 6, 12, 20, 16, 3, 14)_{\Theta(3,3,6)}, (14, 3, 5, 7, 12, 15, 18, 11, 16, 23, 0)_{\Theta(3,3,6)},$
 $(0, 3, 11, 19, 20, 22, 4, 7, 23, 15, 8)_{\Theta(3,3,6)}, (6, 19, 4, 8, 21, 23, 10, 3, 11, 15, 7)_{\Theta(3,3,6)},$
 $(16, 22, 15, 13, 19, 2, 12, 4, 20, 23, 11)_{\Theta(3,3,6)},$
 $(16, 9, 17, 13, 2, 11, 4, 8, 22, 14, 19)_{\Theta(3,4,5)}, (21, 19, 6, 7, 22, 4, 15, 2, 20, 8, 11)_{\Theta(3,4,5)},$
 $(3, 5, 8, 7, 4, 1, 19, 6, 9, 14, 18)_{\Theta(3,4,5)}, (22, 4, 5, 17, 10, 11, 0, 20, 6, 2, 7)_{\Theta(3,4,5)},$
 $(14, 11, 16, 9, 8, 10, 13, 7, 1, 17, 15)_{\Theta(3,4,5)}, (22, 21, 11, 5, 7, 10, 20, 9, 2, 1, 16)_{\Theta(3,4,5)},$
 $(10, 20, 1, 15, 3, 12, 4, 18, 7, 21, 0)_{\Theta(3,4,5)}, (16, 20, 10, 12, 7, 0, 13, 5, 23, 9, 18)_{\Theta(3,4,5)},$

$(2, 16, 10, 23, 15, 7, 13, 4, 8, 21, 12)_{\Theta(3,4,5)}, (7, 15, 12, 5, 17, 2, 8, 23, 13, 4, 21)_{\Theta(3,4,5)},$
 $(18, 23, 0, 15, 2, 19, 4, 11, 20, 5, 8)_{\Theta(3,4,5)},$
 $(20, 6, 21, 4, 11, 14, 18, 17, 2, 10, 5)_{\Theta(4,4,4)}, (13, 12, 10, 14, 22, 17, 16, 0, 2, 8, 11)_{\Theta(4,4,4)},$
 $(5, 19, 7, 4, 2, 8, 22, 0, 17, 15, 18)_{\Theta(4,4,4)}, (8, 1, 14, 15, 22, 13, 21, 3, 7, 2, 16)_{\Theta(4,4,4)},$
 $(5, 21, 14, 12, 8, 19, 3, 15, 20, 0, 7)_{\Theta(4,4,4)}, (21, 7, 14, 9, 3, 19, 8, 1, 17, 20, 15)_{\Theta(4,4,4)},$
 $(20, 11, 10, 22, 7, 4, 17, 1, 9, 2, 14)_{\Theta(4,4,4)}, (11, 18, 2, 15, 1, 20, 7, 16, 22, 19, 6)_{\Theta(4,4,4)},$
 $(2, 4, 0, 15, 19, 12, 23, 9, 17, 7, 18)_{\Theta(4,4,4)}, (3, 10, 12, 20, 1, 14, 23, 8, 17, 9, 19)_{\Theta(4,4,4)},$
 $(3, 12, 6, 15, 4, 18, 9, 1, 23, 10, 17)_{\Theta(4,4,4)}$

under the action of the mapping $x \mapsto x + 8 \pmod{24}$ for only the first six graphs in each design. \square

Lemma 3.6 *There exist decompositions into $\Theta(a, b, c)$ of the complete bipartite graph $K_{12,8}$ for $a + b + c = 12$ with a, b, c all even. There exist decompositions into $\Theta(a, b, c)$ of the complete multipartite graphs $K_{8,8,8}$, $K_{8,8,8,8}$ and $K_{8,8,8,24}$ for $a + b + c = 12$ with a, b, c not all even.*

Proof The decompositions are presented in Appendix C. \square

Theorem 1.4 for $\Theta(a, b, c)$ with $a + b + c = 12$ follows from Theorem 1.1, Lemmas 3.5 and 3.6, and Proposition 2.1 (with $d = 4, r = 3, s = 2, f = 1$ and $g = 0, 2$ or 4) if a, b and c are all even, Proposition 2.6 otherwise. \square

Lemma 3.7 *There exist $\Theta(a, b, c)$ designs of orders 13 and 14 for $a + b + c = 13$. There exists a $\Theta(a, b, c)$ design of order 26 for $a + b + c = 13$ with a, b, c not all odd.*

Proof There are thirteen theta graphs of which four are bipartite. The decompositions are presented in Appendix D. \square

Lemma 3.8 *There exist decompositions into $\Theta(a, b, c)$ of the complete bipartite graph $K_{13,13}$ for $a + b + c = 13$ with a, b, c all odd. There exist decompositions into $\Theta(a, b, c)$ of the complete multipartite graphs $K_{13,13,13}$, $K_{13,13,13,13}$ and $K_{13,13,13,13,13}$ for $a + b + c = 13$ with a, b, c not all odd.*

Proof The decompositions are presented in Appendix D. \square

Theorem 1.5 for $\Theta(a, b, c)$ with $a + b + c = 13$ follows from Theorem 1.1, Lemmas 3.7 and 3.8, and Proposition 2.1 (with $d = 13, r = s = f = 1$ and $g = 0$) if a, b and c are all odd, Proposition 2.3 otherwise. \square

Lemma 3.9 *There exist $\Theta(a, b, c)$ designs of order 21, 28 and 36 for $a + b + c = 14$. There exist $\Theta(a, b, c)$ designs of order 49, 56, 57, 64, 77 and 92 for $a + b + c = 14$ with a, b, c not all even.*

Proof There are fifteen theta graphs of which four are bipartite. The decompositions are presented in Appendix E. \square

Lemma 3.10 *There exist decompositions into $\Theta(a, b, c)$ of the complete bipartite graph $K_{14,7}$ for $a + b + c = 14$ with a, b, c all even. There exist decompositions into $\Theta(a, b, c)$ of the complete multipartite graphs $K_{14,14,14}$, $K_{7,7,7,7}$, $K_{28,28,28,35}$, $K_{7,7,7,7,7}$, $K_{7,7,7,7,21}$ and $K_{7,7,7,7,28}$ for $a + b + c = 14$ with a, b, c not all even.*

Proof The decompositions are presented in Appendix E. \square

Theorem 1.6 for $\Theta(a, b, c)$ with $a + b + c = 14$ follows from Theorem 1.1, Lemmas 3.9 and 3.10, and Proposition 2.1 (with $d = 7$, $r = 2$, $s = 1$, $f = 2$ and $g = 0, 3$ or 5) if a, b and c are all even, Proposition 2.7 otherwise. \square

Lemma 3.11 *There exist $\Theta(a, b, c)$ designs of orders 15, 16, 21, and 25 for $a + b + c = 15$. There exist $\Theta(a, b, c)$ designs of orders 30, 36, 40, 51, 55, 66 and 70 for $a + b + c = 15$ with a, b, c not all odd.*

Proof There are eighteen 15-edge theta graphs of which six are bipartite. We present designs of order 30 here; the remaining decompositions are presented in Appendix F.

K_{30} Let the vertex set be Z_{30} . The decompositions consist of the graphs

- $(6, 2, 0, 7, 22, 5, 19, 15, 18, 25, 23, 1, 21, 12)_{\Theta(1,2,12)},$
- $(7, 20, 15, 2, 10, 9, 8, 5, 6, 17, 3, 18, 28, 22)_{\Theta(1,2,12)},$
- $(20, 4, 14, 19, 1, 3, 5, 18, 11, 16, 24, 13, 2, 21)_{\Theta(1,2,12)},$
- $(7, 13, 10, 14, 6, 11, 8, 4, 25, 9, 18, 21, 27, 23)_{\Theta(1,2,12)},$
- $(27, 5, 16, 22, 6, 18, 13, 0, 14, 2, 9, 21, 11, 4)_{\Theta(1,2,12)},$
- $(5, 17, 26, 11, 23, 2, 15, 22, 4, 6, 10, 28, 21, 8)_{\Theta(1,2,12)},$
- $(23, 5, 14, 17, 2, 11, 7, 28, 24, 22, 18, 16, 25, 29)_{\Theta(1,2,12)},$
- $(17, 29, 11, 13, 4, 0, 28, 16, 12, 10, 22, 1, 5, 20)_{\Theta(1,2,12)},$
- $(23, 29, 8, 19, 10, 3, 20, 11, 26, 9, 16, 4, 27, 14)_{\Theta(1,2,12)},$
- $(19, 1, 6, 11, 23, 0, 27, 21, 5, 18, 14, 7, 15, 17)_{\Theta(1,4,10)},$
- $(15, 19, 0, 28, 20, 1, 6, 9, 18, 22, 11, 16, 25, 8)_{\Theta(1,4,10)},$
- $(17, 26, 16, 15, 8, 23, 10, 28, 5, 9, 20, 12, 19, 21)_{\Theta(1,4,10)},$
- $(0, 9, 1, 10, 21, 14, 12, 23, 15, 25, 5, 7, 2, 26)_{\Theta(1,4,10)},$
- $(10, 15, 4, 2, 28, 25, 19, 16, 13, 17, 27, 26, 6, 22)_{\Theta(1,4,10)},$
- $(12, 0, 5, 20, 23, 24, 4, 14, 11, 2, 16, 6, 28, 18)_{\Theta(1,4,10)},$
- $(12, 18, 4, 20, 10, 11, 26, 23, 14, 29, 6, 5, 8, 17)_{\Theta(1,4,10)},$
- $(0, 24, 10, 26, 16, 29, 2, 17, 14, 28, 8, 22, 12, 6)_{\Theta(1,4,10)},$
- $(18, 24, 11, 8, 23, 6, 0, 22, 2, 5, 26, 29, 20, 17)_{\Theta(1,4,10)},$
- $(5, 2, 26, 10, 18, 8, 4, 16, 25, 14, 20, 17, 21, 6)_{\Theta(1,6,8)},$
- $(9, 21, 18, 15, 4, 17, 22, 11, 12, 24, 13, 10, 0, 14)_{\Theta(1,6,8)},$
- $(10, 6, 15, 2, 3, 27, 13, 11, 17, 0, 8, 23, 12, 7)_{\Theta(1,6,8)},$
- $(24, 0, 22, 19, 15, 20, 9, 26, 14, 13, 8, 28, 12, 5)_{\Theta(1,6,8)},$
- $(21, 13, 19, 28, 4, 26, 17, 18, 1, 14, 7, 5, 15, 23)_{\Theta(1,6,8)},$
- $(19, 7, 5, 17, 3, 10, 25, 11, 4, 22, 15, 28, 13, 1)_{\Theta(1,6,8)},$
- $(7, 23, 13, 3, 16, 28, 5, 27, 10, 22, 29, 15, 25, 11)_{\Theta(1,6,8)},$
- $(1, 19, 17, 10, 28, 21, 4, 16, 9, 22, 7, 29, 13, 25)_{\Theta(1,6,8)},$
- $(1, 23, 21, 5, 13, 19, 9, 25, 17, 29, 11, 27, 4, 16)_{\Theta(1,6,8)},$
- $(12, 18, 17, 28, 24, 15, 7, 19, 22, 3, 23, 9, 27, 2)_{\Theta(2,2,11)},$
- $(18, 8, 15, 7, 5, 2, 24, 9, 22, 1, 27, 28, 11, 4)_{\Theta(2,2,11)},$
- $(15, 22, 17, 10, 11, 19, 23, 8, 6, 12, 13, 26, 20, 2)_{\Theta(2,2,11)},$
- $(15, 13, 23, 22, 25, 14, 28, 6, 9, 0, 4, 26, 17, 20)_{\Theta(2,2,11)},$
- $(5, 21, 7, 19, 23, 22, 20, 0, 25, 12, 8, 9, 15, 2)_{\Theta(2,2,11)},$
- $(13, 16, 19, 10, 6, 17, 24, 5, 26, 1, 7, 0, 28, 22)_{\Theta(2,2,11)},$

$(4, 25, 10, 19, 23, 17, 28, 1, 24, 22, 11, 18, 29, 20)_{\Theta(2,2,11)},$
 $(4, 13, 7, 28, 6, 29, 23, 14, 19, 12, 5, 11, 17, 8)_{\Theta(2,2,11)},$
 $(16, 25, 1, 18, 5, 29, 10, 12, 23, 0, 11, 2, 7, 22)_{\Theta(2,2,11)},$
 $(12, 19, 17, 15, 23, 14, 4, 21, 1, 25, 5, 2, 6, 3)_{\Theta(2,3,10)},$
 $(22, 2, 1, 6, 17, 25, 28, 11, 4, 0, 18, 10, 16, 27)_{\Theta(2,3,10)},$
 $(6, 0, 21, 20, 28, 12, 2, 25, 11, 15, 10, 14, 26, 17)_{\Theta(2,3,10)},$
 $(11, 16, 17, 2, 9, 12, 5, 23, 28, 26, 15, 6, 13, 1)_{\Theta(2,3,10)},$
 $(22, 9, 10, 12, 1, 11, 21, 23, 26, 20, 4, 25, 0, 8)_{\Theta(2,3,10)},$
 $(1, 13, 0, 27, 9, 18, 19, 2, 7, 20, 25, 8, 21, 15)_{\Theta(2,3,10)},$
 $(3, 1, 20, 7, 26, 15, 19, 8, 13, 5, 21, 25, 27, 14)_{\Theta(2,3,10)},$
 $(3, 9, 21, 1, 23, 17, 25, 14, 19, 11, 27, 15, 29, 7)_{\Theta(2,3,10)},$
 $(9, 13, 26, 15, 2, 3, 27, 21, 19, 6, 7, 24, 25, 12)_{\Theta(2,3,10)},$
 $(19, 3, 9, 0, 22, 7, 6, 20, 26, 10, 28, 17, 4, 18)_{\Theta(2,4,9)},$
 $(14, 6, 4, 26, 19, 13, 9, 20, 7, 8, 16, 12, 11, 24)_{\Theta(2,4,9)},$
 $(8, 28, 15, 18, 26, 5, 21, 17, 19, 14, 25, 4, 7, 23)_{\Theta(2,4,9)},$
 $(11, 20, 23, 7, 28, 21, 17, 27, 6, 15, 16, 22, 3, 5)_{\Theta(2,4,9)},$
 $(10, 4, 8, 11, 3, 1, 15, 12, 23, 9, 21, 7, 29, 24)_{\Theta(2,4,9)},$
 $(1, 25, 0, 11, 20, 24, 19, 7, 17, 26, 23, 2, 6, 3)_{\Theta(2,4,9)},$
 $(25, 18, 13, 5, 14, 11, 7, 12, 6, 29, 2, 0, 24, 19)_{\Theta(2,4,9)},$
 $(1, 12, 9, 6, 8, 5, 13, 23, 0, 26, 24, 17, 20, 18)_{\Theta(2,4,9)},$
 $(12, 18, 14, 13, 21, 24, 8, 29, 19, 27, 0, 6, 7, 15)_{\Theta(2,4,9)},$
 $(5, 15, 17, 7, 4, 20, 28, 16, 26, 25, 9, 27, 18, 2)_{\Theta(2,5,8)},$
 $(25, 27, 24, 6, 26, 5, 1, 3, 2, 8, 11, 18, 10, 7)_{\Theta(2,5,8)},$
 $(11, 24, 4, 0, 12, 6, 22, 19, 14, 18, 17, 26, 8, 1)_{\Theta(2,5,8)},$
 $(26, 28, 21, 7, 9, 15, 24, 3, 6, 14, 12, 25, 13, 19)_{\Theta(2,5,8)},$
 $(5, 3, 18, 22, 27, 28, 23, 0, 25, 10, 19, 2, 17, 11)_{\Theta(2,5,8)},$
 $(5, 11, 25, 9, 28, 22, 10, 21, 2, 4, 8, 27, 16, 14)_{\Theta(2,5,8)},$
 $(10, 17, 21, 14, 3, 22, 16, 4, 28, 29, 15, 26, 23, 7)_{\Theta(2,5,8)},$
 $(5, 10, 8, 4, 22, 26, 28, 19, 29, 13, 23, 9, 20, 16)_{\Theta(2,5,8)},$
 $(11, 17, 1, 27, 23, 22, 20, 15, 4, 16, 28, 2, 29, 3)_{\Theta(2,5,8)},$
 $(4, 7, 25, 26, 27, 15, 23, 10, 9, 12, 28, 14, 16, 22)_{\Theta(2,6,7)},$
 $(26, 12, 16, 9, 19, 11, 8, 1, 5, 6, 24, 18, 27, 13)_{\Theta(2,6,7)},$
 $(3, 5, 14, 22, 1, 20, 26, 11, 7, 17, 12, 2, 9, 10)_{\Theta(2,6,7)},$
 $(13, 0, 11, 8, 5, 23, 27, 2, 18, 14, 26, 22, 4, 7)_{\Theta(2,6,7)},$
 $(7, 18, 1, 8, 24, 2, 19, 5, 11, 10, 17, 27, 25, 3)_{\Theta(2,6,7)},$
 $(21, 22, 0, 15, 12, 4, 27, 24, 28, 17, 3, 5, 16, 9)_{\Theta(2,6,7)},$
 $(22, 27, 11, 12, 3, 16, 18, 10, 15, 28, 6, 4, 23, 21)_{\Theta(2,6,7)},$
 $(9, 21, 18, 6, 29, 15, 24, 4, 23, 0, 3, 10, 12, 5)_{\Theta(2,6,7)},$
 $(9, 27, 3, 15, 17, 24, 16, 6, 11, 18, 28, 0, 10, 29)_{\Theta(2,6,7)},$
 $(19, 17, 3, 15, 13, 16, 9, 23, 7, 24, 12, 14, 21, 2)_{\Theta(3,4,8)},$
 $(11, 25, 16, 17, 18, 24, 2, 1, 19, 28, 8, 3, 6, 10)_{\Theta(3,4,8)},$
 $(4, 14, 9, 10, 15, 11, 20, 2, 5, 7, 26, 1, 6, 27)_{\Theta(3,4,8)},$
 $(6, 15, 26, 14, 17, 0, 9, 2, 19, 18, 7, 4, 12, 28)_{\Theta(3,4,8)},$
 $(11, 19, 22, 12, 21, 6, 9, 27, 1, 3, 25, 16, 29, 20)_{\Theta(3,4,8)},$
 $(22, 11, 16, 4, 28, 20, 17, 10, 12, 26, 23, 24, 8, 5)_{\Theta(3,4,8)},$
 $(10, 22, 16, 8, 17, 23, 29, 26, 4, 6, 5, 0, 28, 14)_{\Theta(3,4,8)},$

$(10, 29, 28, 5, 4, 22, 24, 2, 16, 23, 18, 17, 12, 11)_{\Theta(3,4,8)},$
 $(11, 29, 14, 0, 23, 5, 17, 6, 20, 4, 28, 16, 18, 2)_{\Theta(3,4,8)},$
 $(21, 6, 10, 11, 5, 2, 16, 3, 8, 24, 17, 27, 23, 4)_{\Theta(3,6,6)},$
 $(2, 6, 18, 25, 7, 27, 8, 14, 26, 19, 4, 28, 24, 15)_{\Theta(3,6,6)},$
 $(11, 16, 4, 8, 0, 17, 25, 22, 6, 23, 15, 10, 7, 26)_{\Theta(3,6,6)},$
 $(4, 22, 17, 13, 16, 14, 27, 24, 0, 3, 5, 10, 19, 15)_{\Theta(3,6,6)},$
 $(7, 24, 8, 9, 19, 18, 0, 13, 20, 12, 11, 26, 23, 2)_{\Theta(3,6,6)},$
 $(1, 9, 17, 7, 3, 15, 27, 19, 25, 11, 13, 23, 14, 21)_{\Theta(3,6,6)},$
 $(25, 1, 5, 29, 11, 17, 8, 15, 7, 27, 13, 21, 3, 9)_{\Theta(3,6,6)},$
 $(9, 19, 27, 21, 2, 11, 5, 26, 3, 15, 13, 7, 23, 29)_{\Theta(3,6,6)},$
 $(19, 25, 17, 23, 5, 7, 21, 15, 1, 13, 29, 20, 27, 3)_{\Theta(3,6,6)},$
 $(19, 9, 5, 1, 12, 26, 15, 22, 16, 10, 23, 21, 7, 28)_{\Theta(4,4,7)},$
 $(13, 3, 2, 28, 18, 19, 24, 7, 14, 5, 10, 25, 23, 15)_{\Theta(4,4,7)},$
 $(10, 20, 12, 3, 25, 22, 11, 15, 9, 4, 0, 1, 21, 5)_{\Theta(4,4,7)},$
 $(9, 23, 7, 20, 18, 26, 4, 12, 0, 24, 6, 22, 2, 3)_{\Theta(4,4,7)},$
 $(6, 1, 20, 17, 23, 13, 22, 19, 14, 8, 15, 21, 18, 11)_{\Theta(4,4,7)},$
 $(11, 14, 24, 23, 16, 8, 10, 26, 12, 2, 6, 5, 18, 17)_{\Theta(4,4,7)},$
 $(14, 20, 23, 22, 29, 28, 26, 0, 18, 8, 17, 5, 2, 4)_{\Theta(4,4,7)},$
 $(11, 29, 10, 17, 0, 23, 5, 28, 2, 14, 24, 20, 8, 26)_{\Theta(4,4,7)},$
 $(20, 29, 2, 16, 17, 22, 8, 12, 23, 6, 26, 5, 4, 11)_{\Theta(4,4,7)},$
 $(11, 7, 6, 8, 15, 18, 4, 9, 19, 28, 27, 0, 10, 26)_{\Theta(4,5,6)},$
 $(24, 28, 2, 17, 19, 5, 14, 13, 10, 25, 16, 3, 8, 21)_{\Theta(4,5,6)},$
 $(26, 9, 28, 5, 21, 8, 16, 27, 13, 22, 17, 18, 2, 11)_{\Theta(4,5,6)},$
 $(1, 22, 24, 27, 18, 5, 9, 7, 23, 25, 12, 4, 6, 2)_{\Theta(4,5,6)},$
 $(28, 13, 25, 20, 26, 22, 7, 14, 24, 17, 9, 8, 11, 23)_{\Theta(4,5,6)},$
 $(23, 9, 1, 6, 24, 17, 11, 14, 3, 26, 15, 0, 12, 18)_{\Theta(4,5,6)},$
 $(0, 9, 25, 17, 20, 6, 27, 12, 29, 18, 24, 11, 21, 15)_{\Theta(4,5,6)},$
 $(0, 5, 24, 19, 11, 17, 27, 3, 18, 21, 6, 12, 7, 29)_{\Theta(4,5,6)},$
 $(5, 23, 13, 18, 6, 15, 24, 12, 3, 8, 27, 21, 2, 29)_{\Theta(4,5,6)}$

under the action of the mapping $x \mapsto x + 6 \pmod{30}$ for only the first five graphs in each design. \square

Lemma 3.12 *There exist decompositions into $\Theta(a, b, c)$ of the complete bipartite graphs $K_{15,15}$, $K_{15,20}$ and $K_{15,25}$ for $a+b+c = 15$ with a, b, c all odd. There exist decompositions into $\Theta(a, b, c)$ of the complete multipartite graphs $K_{5,5,5}$ and $K_{5,5,5,5}$ for $a+b+c = 15$ with a, b, c not all odd.*

Proof The decompositions are presented in Appendix F. \square

Theorem 1.7 follows from Theorem 1.1, Lemmas 3.11 and 3.12, and Proposition 2.2 (with $(r, s, e) = (15, 15, 0), (15, 15, 1), (15, 20, 1)$ or $(15, 25, 0)$) if a, b and c are all odd, Proposition 2.8 otherwise. \square

References

- [1] R. J. R. Abel, F. E. Bennett and M. Greig, PBD-Closure, *Handbook of Combinatorial Designs*, second edition (ed. C. J. Colbourn and J. H. Dinitz), Chapman & Hall/CRC

Press (2007), 247–255.

- [2] P. Adams, D. E. Bryant and M. Buchanan, A survey on the existence of G -designs, *J. Combin. Des.* **16** (2008), 373–410.
- [3] A. Blinco, On diagonal cycle systems, *Australas. J. Combin.* **24** (2001), 221–230.
- [4] A. Blinco, Decompositions of complete graphs into theta graphs with fewer than ten edges, *Utilitas Mathematica* **64** (2003), 197–212.
- [5] D. E. Bryant and T. A. McCourt. Existence results for G -designs, <http://wiki.smp.uq.edu.au/G-designs/>.
- [6] A. D. Forbes. Solution 265.6 – Triples, *M500* **276** (June 2017).
- [7] G. Ge, Group divisible designs, *Handbook of Combinatorial Designs* second edition (ed. C. J. Colbourn and J. H. Dinitz), Chapman & Hall/CRC Press (2007), 255–260.
- [8] G. Ge and A. C. H. Ling, Asymptotic results on the existence of 4-RGDDs and uniform 5-GDDs, *J. Combin. Des.* **13** (2005) 222–237.
- [9] G. Ge and Y. Miao, PBDs, Frames and Resolvability, *Handbook of Combinatorial Designs*, second edition (ed. C. J. Colbourn and J. H. Dinitz), Chapman & Hall/CRC Press (2007), 261–265.
- [10] N. Punnim and N. Pabhapote. On graceful graphs: cycles with a P_k chord, $k \geq 4$, *Ars Combin.* **23A** (1987), 225–228.
- [11] R. S. Rees, Two New Direct Product-Type Constructions for Resolvable Group-Divisible Designs, *J. Combin. Designs* **1** (1993), 15–26.
- [12] R. M. Wilson, An existence theory for pairwise balanced designs I: Composition theorems and morphisms, *J. Combin. Theory Ser. A*, **13** (1971), 220–245.

A Theta graphs of 10 edges

Proof of Lemma 3.1

K_{16} Let the vertex set be $Z_{15} \cup \{\infty\}$. The decompositions consist of the graphs

$$\begin{aligned}
 &(\infty, 11, 5, 9, 2, 10, 8, 7, 3)_{\Theta(1,2,7)}, \\
 &(11, 12, 14, 0, 2, 6, 3, 8, 9)_{\Theta(1,2,7)}, \\
 &(14, 1, 3, 0, 4, 10, 7, 2, 11)_{\Theta(1,2,7)}, \\
 &(0, 3, 10, 1, 9, 4, 13, 7, \infty)_{\Theta(1,2,7)}, \\
 &(\infty, 7, 10, 1, 13, 14, 4, 2, 0)_{\Theta(1,3,6)}, \\
 &(1, 8, 4, 12, 2, 3, 9, 5, 0)_{\Theta(1,3,6)}, \\
 &(8, 3, 10, 6, 2, 5, 4, 11, 1)_{\Theta(1,3,6)}, \\
 &(0, 9, 1, \infty, 3, 14, 2, 7, 11)_{\Theta(1,3,6)}, \\
 &(\infty, 5, 8, 14, 12, 1, 11, 3, 6)_{\Theta(1,4,5)}, \\
 &(10, 7, 4, 0, 13, 1, 14, 9, 6)_{\Theta(1,4,5)}, \\
 &(13, 14, 9, 1, 7, 3, 0, 2, \infty)_{\Theta(1,4,5)}, \\
 &(0, 11, 10, 3, 7, 14, 2, 12, 13)_{\Theta(1,4,5)}, \\
 &(\infty, 6, 3, 11, 4, 14, 10, 1, 5)_{\Theta(2,2,6)}, \\
 &(4, 5, 13, 12, 2, 14, 6, 9, 8)_{\Theta(2,2,6)}, \\
 &(0, 8, 12, 2, 5, 14, 3, 11, 13)_{\Theta(2,2,6)}, \\
 &(0, 12, 13, \infty, 14, 1, 2, 11, 7)_{\Theta(2,2,6)}, \\
 &(\infty, 12, 6, 8, 4, 14, 11, 3, 10)_{\Theta(2,3,5)}, \\
 &(4, 0, 3, 11, 10, 9, 12, 1, 13)_{\Theta(2,3,5)}, \\
 &(9, 14, 10, 0, 12, 3, 7, 2, 1)_{\Theta(2,3,5)}, \\
 &(1, 8, 3, 10, 2, 11, 0, \infty, 7)_{\Theta(2,3,5)}, \\
 &(\infty, 14, 11, 7, 8, 10, 4, 5, 12)_{\Theta(2,4,4)}, \\
 &(3, 7, 13, 0, 11, 12, 9, 14, 1)_{\Theta(2,4,4)}, \\
 &(2, 0, 6, 5, 13, 1, 14, 3, \infty)_{\Theta(2,4,4)}, \\
 &(1, 7, 3, 6, 13, 14, 9, 0, 5)_{\Theta(2,4,4)}, \\
 &(\infty, 5, 7, 9, 8, 2, 6, 1, 0)_{\Theta(3,3,4)}, \\
 &(6, 13, 14, 3, 10, 1, 12, 5, 4)_{\Theta(3,3,4)}, \\
 &(13, 9, 6, 2, 0, \infty, 12, 7, 4)_{\Theta(3,3,4)}, \\
 &(2, 8, 0, 9, 13, 5, 1, 4, 6)_{\Theta(3,3,4)}
 \end{aligned}$$

under the action of the mapping $x \mapsto x + 5 \pmod{15}$, $\infty \mapsto \infty$.

K_{25} Let the vertex set be Z_{25} . The decompositions consist of the graphs

$$\begin{aligned}
 &(9, 10, 6, 11, 19, 22, 20, 7, 8)_{\Theta(1,2,7)}, \\
 &(21, 20, 5, 3, 12, 1, 6, 23, 9)_{\Theta(1,2,7)}, \\
 &(11, 4, 2, 21, 8, 0, 3, 1, 14)_{\Theta(1,2,7)}, \\
 &(0, 4, 9, 5, 2, 1, 7, 3, 8)_{\Theta(1,2,7)}, \\
 &(0, 7, 11, 6, 3, 4, 10, 2, 12)_{\Theta(1,2,7)}, \\
 &(2, 14, 8, 9, 17, 3, 18, 5, 23)_{\Theta(1,2,7)}, \\
 &(0, 4, 22, 8, 1, 2, 3, 14, 6)_{\Theta(1,3,6)}, \\
 &(14, 7, 8, 2, 9, 15, 3, 10, 0)_{\Theta(1,3,6)}, \\
 &(13, 14, 1, 12, 21, 9, 6, 0, 5)_{\Theta(1,3,6)}, \\
 &(8, 12, 6, 22, 18, 15, 17, 11, 0)_{\Theta(1,3,6)}, \\
 &(13, 4, 22, 19, 8, 10, 6, 16, 12)_{\Theta(1,3,6)},
 \end{aligned}$$

$(0, 8, 16, 11, 14, 15, 7, 19, 1)_{\Theta(1,3,6)},$
 $(0, 18, 21, 15, 14, 8, 11, 20, 7)_{\Theta(1,4,5)},$
 $(2, 3, 22, 11, 9, 19, 17, 21, 14)_{\Theta(1,4,5)},$
 $(19, 9, 14, 5, 15, 7, 3, 22, 21)_{\Theta(1,4,5)},$
 $(14, 23, 10, 17, 20, 0, 1, 18, 11)_{\Theta(1,4,5)},$
 $(15, 3, 7, 23, 8, 13, 14, 11, 1)_{\Theta(1,4,5)},$
 $(2, 24, 0, 5, 16, 12, 6, 1, 17)_{\Theta(1,4,5)},$
 $(0, 8, 13, 19, 7, 10, 9, 20, 21)_{\Theta(2,2,6)},$
 $(20, 6, 1, 23, 12, 2, 19, 7, 9)_{\Theta(2,2,6)},$
 $(20, 3, 4, 18, 6, 15, 5, 0, 2)_{\Theta(2,2,6)},$
 $(5, 23, 1, 17, 13, 4, 0, 18, 2)_{\Theta(2,2,6)},$
 $(9, 18, 11, 16, 13, 19, 14, 4, 7)_{\Theta(2,2,6)},$
 $(1, 4, 16, 22, 7, 6, 17, 12, 21)_{\Theta(2,2,6)},$
 $(0, 9, 15, 8, 6, 21, 5, 12, 7)_{\Theta(2,3,5)},$
 $(2, 7, 18, 21, 20, 11, 0, 5, 3)_{\Theta(2,3,5)},$
 $(16, 23, 8, 9, 3, 11, 19, 15, 14)_{\Theta(2,3,5)},$
 $(4, 23, 19, 3, 16, 9, 11, 21, 17)_{\Theta(2,3,5)},$
 $(12, 2, 19, 9, 0, 22, 11, 8, 5)_{\Theta(2,3,5)},$
 $(0, 16, 17, 18, 4, 13, 12, 24, 10)_{\Theta(2,3,5)},$
 $(0, 14, 11, 18, 23, 13, 7, 1, 10)_{\Theta(2,4,4)},$
 $(22, 8, 0, 19, 21, 20, 7, 5, 17)_{\Theta(2,4,4)},$
 $(22, 20, 9, 8, 19, 23, 2, 4, 10)_{\Theta(2,4,4)},$
 $(4, 19, 9, 22, 18, 1, 5, 0, 6)_{\Theta(2,4,4)},$
 $(9, 1, 3, 0, 23, 22, 17, 6, 13)_{\Theta(2,4,4)},$
 $(1, 8, 11, 6, 10, 2, 17, 16, 24)_{\Theta(2,4,4)},$
 $(0, 9, 22, 7, 10, 14, 18, 6, 12)_{\Theta(3,3,4)},$
 $(22, 13, 23, 14, 8, 3, 17, 16, 9)_{\Theta(3,3,4)},$
 $(3, 5, 21, 18, 9, 6, 12, 10, 17)_{\Theta(3,3,4)},$
 $(8, 16, 2, 6, 19, 20, 10, 5, 21)_{\Theta(3,3,4)},$
 $(14, 18, 5, 22, 16, 10, 4, 15, 1)_{\Theta(3,3,4)},$
 $(1, 7, 9, 21, 17, 24, 3, 0, 19)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 5 \pmod{25}$.

K_{36} Let the vertex set be Z_{36} . The decompositions consist of the graphs

$(2, 3, 8, 17, 24, 30, 25, 29, 18)_{\Theta(1,2,7)},$
 $(5, 29, 31, 20, 7, 11, 34, 17, 16)_{\Theta(1,2,7)},$
 $(12, 28, 31, 10, 24, 20, 34, 6, 18)_{\Theta(1,2,7)},$
 $(2, 7, 29, 27, 20, 11, 19, 31, 21)_{\Theta(1,2,7)},$
 $(20, 8, 19, 12, 14, 13, 4, 9, 25)_{\Theta(1,2,7)},$
 $(0, 10, 33, 15, 6, 2, 5, 11, 17)_{\Theta(1,2,7)},$
 $(1, 19, 35, 29, 4, 22, 3, 6, 26)_{\Theta(1,2,7)},$
 $(17, 7, 20, 31, 33, 26, 25, 12, 15)_{\Theta(1,3,6)},$
 $(0, 28, 14, 12, 4, 33, 19, 2, 26)_{\Theta(1,3,6)},$
 $(1, 14, 15, 8, 7, 12, 13, 24, 29)_{\Theta(1,3,6)},$
 $(14, 20, 4, 3, 5, 1, 9, 33, 2)_{\Theta(1,3,6)},$
 $(28, 16, 13, 7, 15, 33, 31, 22, 2)_{\Theta(1,3,6)},$

$(23, 10, 26, 11, 19, 17, 34, 6, 35)_{\Theta(1,3,6)},$
 $(1, 11, 26, 31, 34, 30, 4, 13, 32)_{\Theta(1,3,6)},$
 $(0, 15, 12, 25, 31, 2, 28, 18, 21)_{\Theta(1,4,5)},$
 $(0, 8, 6, 17, 7, 32, 21, 2, 9)_{\Theta(1,4,5)},$
 $(31, 21, 29, 2, 26, 24, 4, 23, 34)_{\Theta(1,4,5)},$
 $(1, 19, 32, 29, 15, 13, 14, 30, 34)_{\Theta(1,4,5)},$
 $(9, 28, 29, 0, 3, 1, 33, 11, 10)_{\Theta(1,4,5)},$
 $(19, 32, 24, 33, 18, 10, 2, 7, 30)_{\Theta(1,4,5)},$
 $(2, 19, 24, 18, 11, 35, 1, 16, 7)_{\Theta(1,4,5)},$
 $(0, 33, 6, 2, 34, 26, 19, 14, 25)_{\Theta(2,3,5)},$
 $(19, 29, 9, 2, 14, 22, 23, 31, 8)_{\Theta(2,3,5)},$
 $(14, 25, 21, 1, 20, 23, 29, 17, 22)_{\Theta(2,3,5)},$
 $(34, 33, 23, 12, 31, 4, 18, 1, 15)_{\Theta(2,3,5)},$
 $(1, 20, 24, 23, 28, 12, 5, 32, 11)_{\Theta(2,3,5)},$
 $(24, 28, 12, 25, 31, 6, 14, 16, 27)_{\Theta(2,3,5)},$
 $(0, 26, 10, 7, 11, 33, 35, 15, 3)_{\Theta(2,3,5)},$
 $(0, 1, 22, 4, 15, 30, 28, 3, 33)_{\Theta(3,3,4)},$
 $(20, 22, 21, 32, 29, 11, 27, 10, 1)_{\Theta(3,3,4)},$
 $(30, 13, 16, 0, 22, 20, 24, 19, 33)_{\Theta(3,3,4)},$
 $(23, 14, 33, 27, 15, 19, 24, 28, 26)_{\Theta(3,3,4)},$
 $(4, 30, 14, 23, 16, 10, 7, 19, 17)_{\Theta(3,3,4)},$
 $(13, 21, 2, 33, 32, 19, 28, 11, 31)_{\Theta(3,3,4)},$
 $(1, 6, 2, 3, 29, 10, 32, 23, 9)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 4 \pmod{36}$.

K_{40} Let the vertex set be $Z_{39} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 25, 33, 26, 38, 32, 30, 21, 7)_{\Theta(1,2,7)},$
 $(2, 33, 36, 37, 22, 20, 28, 24, 14)_{\Theta(1,2,7)},$
 $(17, 22, 3, 6, 38, 13, 2, 20, 23)_{\Theta(1,2,7)},$
 $(30, 31, 36, 18, 35, 12, 8, 28, 25)_{\Theta(1,2,7)},$
 $(19, 35, 9, 32, 2, 3, 10, 27, 25)_{\Theta(1,2,7)},$
 $(3, 31, 18, 21, 37, 10, 1, 23, 38)_{\Theta(1,2,7)},$
 $(\infty, 38, 28, 29, 33, 3, 16, 8, 9)_{\Theta(1,3,6)},$
 $(27, 12, 0, 26, 16, 33, 11, 3, 13)_{\Theta(1,3,6)},$
 $(22, 35, 34, 1, 6, 29, 17, 13, 33)_{\Theta(1,3,6)},$
 $(16, 25, 31, 27, 14, 8, 22, 38, 18)_{\Theta(1,3,6)},$
 $(23, 20, 34, 13, 2, 9, 6, 12, 37)_{\Theta(1,3,6)},$
 $(38, 3, 14, 34, 27, 6, 1, 37, 8)_{\Theta(1,3,6)},$
 $(\infty, 22, 33, 0, 32, 26, 15, 3, 10)_{\Theta(1,4,5)},$
 $(31, 29, 34, 16, 27, 30, 8, 5, 0)_{\Theta(1,4,5)},$
 $(8, 21, 26, 1, 24, 7, 29, 37, 6)_{\Theta(1,4,5)},$
 $(19, 9, 33, 29, 13, 34, 28, 37, 32)_{\Theta(1,4,5)},$
 $(23, 4, 9, 30, 21, 34, 2, 8, 17)_{\Theta(1,4,5)},$
 $(1, 3, 5, 24, 37, 21, 29, 14, 2)_{\Theta(1,4,5)},$
 $(\infty, 26, 1, 6, 31, 38, 23, 25, 12)_{\Theta(2,3,5)},$
 $(9, 5, 38, 26, 0, 30, 36, 19, 21)_{\Theta(2,3,5)},$

$(14, 20, 13, 25, 10, 32, 33, 34, 0)_{\Theta(2,3,5)},$
 $(28, 27, 24, 37, 16, 25, 38, 2, 0)_{\Theta(2,3,5)},$
 $(10, 13, 29, 18, 33, 22, 6, 38, 3)_{\Theta(2,3,5)},$
 $(33, 5, 1, 3, 14, 2, 19, 25, 17)_{\Theta(2,3,5)},$
 $(\infty, 3, 22, 7, 26, 5, 6, 20, 37)_{\Theta(3,3,4)},$
 $(11, 12, 13, 31, 35, 32, 37, 24, 22)_{\Theta(3,3,4)},$
 $(7, 11, 6, 33, 30, 12, 18, 24, 21)_{\Theta(3,3,4)},$
 $(37, 15, 23, 22, 32, 6, 34, 4, 8)_{\Theta(3,3,4)},$
 $(27, 1, 13, 32, 3, 8, 35, 7, 34)_{\Theta(3,3,4)},$
 $(0, 5, 22, 38, 23, 32, 11, 1, 9)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 3 \pmod{39}$, $\infty \mapsto \infty$.

K_{41} Let the vertex set be Z_{41} . The decompositions consist of the graphs

$(0, 38, 17, 18, 11, 7, 1, 2, 10)_{\Theta(1,2,7)},$
 $(0, 2, 11, 5, 17, 3, 18, 8, 24)_{\Theta(1,2,7)},$
 $(0, 5, 7, 37, 15, 32, 1, 2, 8)_{\Theta(1,3,6)},$
 $(0, 2, 4, 16, 13, 5, 21, 1, 20)_{\Theta(1,3,6)},$
 $(0, 21, 5, 20, 33, 7, 1, 2, 4)_{\Theta(1,4,5)},$
 $(0, 3, 4, 12, 21, 16, 2, 24, 13)_{\Theta(1,4,5)},$
 $(0, 20, 31, 12, 33, 8, 1, 2, 4)_{\Theta(2,3,5)},$
 $(0, 1, 4, 5, 23, 14, 8, 25, 10)_{\Theta(2,3,5)},$
 $(0, 7, 19, 1, 9, 24, 2, 3, 10)_{\Theta(3,3,4)},$
 $(0, 1, 4, 9, 10, 21, 12, 28, 14)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 1 \pmod{41}$.

K_{45} Let the vertex set be Z_{45} . The decompositions consist of the graphs

$(0, 12, 31, 24, 39, 14, 13, 17, 30)_{\Theta(1,2,7)},$
 $(43, 41, 20, 5, 14, 12, 7, 19, 38)_{\Theta(1,2,7)},$
 $(26, 3, 17, 38, 21, 41, 36, 32, 35)_{\Theta(1,2,7)},$
 $(22, 32, 7, 9, 14, 38, 34, 1, 8)_{\Theta(1,2,7)},$
 $(34, 17, 16, 20, 1, 7, 21, 3, 43)_{\Theta(1,2,7)},$
 $(27, 19, 6, 4, 39, 11, 5, 15, 30)_{\Theta(1,2,7)},$
 $(5, 0, 16, 8, 19, 22, 20, 11, 4)_{\Theta(1,2,7)},$
 $(20, 19, 0, 16, 8, 14, 32, 38, 3)_{\Theta(1,2,7)},$
 $(13, 12, 5, 22, 6, 29, 31, 30, 28)_{\Theta(1,2,7)},$
 $(0, 18, 33, 17, 6, 14, 11, 1, 31)_{\Theta(1,2,7)},$
 $(0, 29, 22, 37, 3, 20, 14, 23, 43)_{\Theta(1,2,7)},$
 $(0, 6, 14, 5, 40, 27, 42, 20, 41)_{\Theta(1,3,6)},$
 $(21, 16, 5, 41, 9, 7, 10, 2, 3)_{\Theta(1,3,6)},$
 $(23, 10, 17, 21, 33, 37, 8, 6, 29)_{\Theta(1,3,6)},$
 $(4, 17, 7, 28, 43, 1, 8, 31, 5)_{\Theta(1,3,6)},$
 $(3, 40, 8, 34, 23, 11, 41, 10, 28)_{\Theta(1,3,6)},$
 $(8, 4, 23, 31, 5, 40, 18, 27, 11)_{\Theta(1,3,6)},$
 $(4, 25, 18, 9, 1, 28, 35, 15, 32)_{\Theta(1,3,6)},$
 $(2, 27, 21, 34, 37, 10, 12, 6, 4)_{\Theta(1,3,6)},$
 $(40, 29, 38, 9, 10, 6, 17, 3, 24)_{\Theta(1,3,6)},$
 $(27, 26, 19, 9, 36, 12, 24, 7, 34)_{\Theta(1,3,6)},$

$(0, 28, 44, 29, 4, 38, 21, 7, 2)_{\Theta(1,3,6)},$
 $(0, 13, 10, 42, 21, 24, 22, 8, 31)_{\Theta(1,4,5)},$
 $(38, 0, 35, 32, 9, 33, 16, 13, 30)_{\Theta(1,4,5)},$
 $(22, 2, 12, 11, 43, 29, 25, 26, 20)_{\Theta(1,4,5)},$
 $(16, 35, 12, 38, 15, 26, 33, 13, 34)_{\Theta(1,4,5)},$
 $(35, 4, 8, 0, 29, 40, 12, 42, 30)_{\Theta(1,4,5)},$
 $(7, 39, 41, 22, 38, 34, 31, 6, 21)_{\Theta(1,4,5)},$
 $(37, 42, 21, 10, 6, 29, 18, 34, 3)_{\Theta(1,4,5)},$
 $(7, 5, 8, 17, 38, 18, 3, 13, 39)_{\Theta(1,4,5)},$
 $(35, 21, 29, 14, 4, 6, 37, 34, 26)_{\Theta(1,4,5)},$
 $(43, 31, 4, 16, 40, 0, 22, 34, 29)_{\Theta(1,4,5)},$
 $(1, 3, 14, 21, 44, 7, 15, 22, 39)_{\Theta(1,4,5)},$
 $(0, 21, 41, 11, 38, 34, 5, 43, 33)_{\Theta(2,3,5)},$
 $(25, 40, 26, 15, 34, 23, 27, 37, 12)_{\Theta(2,3,5)},$
 $(28, 12, 26, 41, 34, 10, 22, 19, 17)_{\Theta(2,3,5)},$
 $(41, 42, 33, 29, 3, 31, 22, 40, 16)_{\Theta(2,3,5)},$
 $(19, 21, 5, 8, 4, 11, 33, 34, 22)_{\Theta(2,3,5)},$
 $(32, 17, 21, 19, 34, 25, 27, 5, 18)_{\Theta(2,3,5)},$
 $(15, 9, 0, 14, 38, 35, 16, 10, 18)_{\Theta(2,3,5)},$
 $(15, 39, 19, 10, 33, 12, 23, 43, 29)_{\Theta(2,3,5)},$
 $(30, 28, 21, 22, 43, 13, 10, 34, 31)_{\Theta(2,3,5)},$
 $(36, 37, 21, 31, 29, 4, 22, 35, 23)_{\Theta(2,3,5)},$
 $(2, 4, 9, 32, 26, 28, 23, 7, 31)_{\Theta(2,3,5)},$
 $(0, 3, 36, 24, 38, 41, 11, 37, 12)_{\Theta(3,3,4)},$
 $(13, 42, 29, 11, 18, 28, 24, 19, 9)_{\Theta(3,3,4)},$
 $(39, 19, 2, 37, 10, 40, 5, 9, 26)_{\Theta(3,3,4)},$
 $(33, 7, 34, 36, 21, 6, 14, 18, 30)_{\Theta(3,3,4)},$
 $(25, 3, 30, 39, 17, 21, 1, 40, 2)_{\Theta(3,3,4)},$
 $(30, 42, 10, 0, 3, 23, 38, 27, 4)_{\Theta(3,3,4)},$
 $(8, 1, 16, 41, 40, 23, 6, 19, 4)_{\Theta(3,3,4)},$
 $(23, 20, 6, 16, 29, 21, 7, 1, 37)_{\Theta(3,3,4)},$
 $(27, 9, 15, 34, 42, 40, 0, 3, 7)_{\Theta(3,3,4)},$
 $(15, 41, 13, 28, 31, 7, 1, 20, 19)_{\Theta(3,3,4)},$
 $(0, 24, 32, 27, 39, 7, 23, 17, 38)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 5 \pmod{45}$.

K_{56} Let the vertex set be $Z_{55} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 12, 18, 29, 24, 51, 31, 1, 7)_{\Theta(1,2,7)},$
 $(33, 32, 39, 54, 18, 10, 41, 8, 51)_{\Theta(1,2,7)},$
 $(39, 35, 41, 42, 28, 21, 11, 50, 27)_{\Theta(1,2,7)},$
 $(37, 51, 30, 19, 43, 29, 28, 52, 36)_{\Theta(1,2,7)},$
 $(4, 48, 2, 37, 0, 51, 9, 29, 35)_{\Theta(1,2,7)},$
 $(9, 0, 24, 30, 41, 43, 16, 34, 15)_{\Theta(1,2,7)},$
 $(16, 4, 30, 3, 48, 8, 43, 46, 14)_{\Theta(1,2,7)},$
 $(27, 14, 0, 1, 2, 5, 3, 7, 11)_{\Theta(1,2,7)},$
 $(0, 1, 10, 2, 11, 3, 8, 4, 12)_{\Theta(1,2,7)},$
 $(0, 3, 20, 5, 4, 11, 6, 14, 25)_{\Theta(1,2,7)},$

$(0, 12, 22, 17, 2, 13, 4, 20, 38)_{\Theta(1,2,7)},$
 $(0, 23, 30, 26, 2, 14, 31, 8, 35)_{\Theta(1,2,7)},$
 $(0, 36, \infty, 42, 3, 22, 47, 19, 2)_{\Theta(1,2,7)},$
 $(1, 18, 34, 38, 9, 39, 7, 27, 48)_{\Theta(1,2,7)},$
 $(\infty, 37, 23, 24, 21, 50, 9, 1, 43)_{\Theta(1,3,6)},$
 $(51, 8, 16, 15, 40, 48, 39, 11, 17)_{\Theta(1,3,6)},$
 $(47, 1, 45, 12, 31, 44, 46, 38, 27)_{\Theta(1,3,6)},$
 $(47, 4, 49, 40, 8, 34, 45, 10, 48)_{\Theta(1,3,6)},$
 $(43, 33, 20, 32, 10, 53, 19, 3, 9)_{\Theta(1,3,6)},$
 $(6, 47, 13, 11, 43, 25, 40, 9, 5)_{\Theta(1,3,6)},$
 $(38, 25, 7, 4, 2, 40, 21, 15, 52)_{\Theta(1,3,6)},$
 $(4, 39, 37, 2, 1, 5, 0, 3, 6)_{\Theta(1,3,6)},$
 $(0, 7, 10, 2, 16, 1, 6, 15, 14)_{\Theta(1,3,6)},$
 $(0, 21, 25, 9, 28, 1, 2, 5, 46)_{\Theta(1,3,6)},$
 $(0, 29, 31, 14, 32, 1, 11, 4, 12)_{\Theta(1,3,6)},$
 $(0, 49, 53, 13, \infty, 4, 8, 3, 17)_{\Theta(1,3,6)},$
 $(1, 18, 19, 47, 22, 32, 2, 17, 38)_{\Theta(1,3,6)},$
 $(3, 28, 36, 32, 26, 49, 19, 9, 14)_{\Theta(1,3,6)},$
 $(\infty, 7, 54, 36, 25, 41, 14, 26, 17)_{\Theta(1,4,5)},$
 $(25, 39, 45, 44, 3, 34, 9, 22, 27)_{\Theta(1,4,5)},$
 $(42, 22, 48, 28, 47, 6, 9, 31, 43)_{\Theta(1,4,5)},$
 $(52, 11, 23, 10, 3, 44, 12, 27, 21)_{\Theta(1,4,5)},$
 $(4, 6, 37, 54, 25, 3, 47, 5, 8)_{\Theta(1,4,5)},$
 $(32, 33, 5, 48, 4, 1, 10, 22, 51)_{\Theta(1,4,5)},$
 $(22, 18, 4, 38, 11, 13, 53, 30, 9)_{\Theta(1,4,5)},$
 $(46, 13, 43, 18, 8, 11, 12, 15, 36)_{\Theta(1,4,5)},$
 $(0, 1, 49, 4, 40, 22, 8, 21, 16)_{\Theta(1,4,5)},$
 $(42, 31, 14, 6, 44, 50, 2, 41, 45)_{\Theta(1,4,5)},$
 $(38, 21, 0, 32, 15, 54, 14, 17, 51)_{\Theta(1,4,5)},$
 $(35, 13, 20, 38, 44, 43, 27, 29, 9)_{\Theta(1,4,5)},$
 $(4, 35, 20, 9, 5, 11, 7, 38, 40)_{\Theta(1,4,5)},$
 $(0, 26, 28, \infty, 50, 45, 47, 54, 49)_{\Theta(1,4,5)},$
 $(\infty, 28, 1, 44, 41, 17, 14, 32, 36)_{\Theta(2,3,5)},$
 $(38, 24, 49, 19, 53, 23, 52, 16, 47)_{\Theta(2,3,5)},$
 $(53, 15, 47, 12, 22, 3, 36, 4, 39)_{\Theta(2,3,5)},$
 $(53, 7, 29, 14, 18, 45, 54, 1, 2)_{\Theta(2,3,5)},$
 $(46, 4, 15, 52, 41, 30, 40, 42, 5)_{\Theta(2,3,5)},$
 $(26, 48, 31, 41, 15, 23, 42, 2, 10)_{\Theta(2,3,5)},$
 $(49, 24, 34, 10, 30, 44, 25, 31, 41)_{\Theta(2,3,5)},$
 $(13, 12, 40, 50, 54, 4, 30, 21, 41)_{\Theta(2,3,5)},$
 $(34, 30, 0, 7, 27, 33, 23, 35, 47)_{\Theta(2,3,5)},$
 $(49, 52, 43, 16, 28, 2, 32, 34, 6)_{\Theta(2,3,5)},$
 $(11, 49, 32, 18, 36, 25, 20, 21, 37)_{\Theta(2,3,5)},$
 $(32, 38, 45, 48, 13, 53, 40, 36, 6)_{\Theta(2,3,5)},$
 $(46, 45, 48, 34, 26, 32, 33, \infty, 5)_{\Theta(2,3,5)},$
 $(1, 12, 45, 49, 8, 35, 3, 5, 19)_{\Theta(2,3,5)},$
 $(\infty, 25, 28, 2, 12, 52, 5, 24, 10)_{\Theta(3,3,4)},$

$(41, 5, 9, 34, 48, 1, 17, 23, 47)_{\Theta(3,3,4)},$
 $(27, 38, 10, 1, 23, 33, 2, 7, 52)_{\Theta(3,3,4)},$
 $(28, 8, 17, 5, 50, 19, 53, 1, 23)_{\Theta(3,3,4)},$
 $(45, 28, 54, 14, 39, 26, 47, 34, 7)_{\Theta(3,3,4)},$
 $(20, 34, 30, 18, 38, 14, 25, 45, 29)_{\Theta(3,3,4)},$
 $(35, 50, 6, 25, 14, 16, 48, 44, 1)_{\Theta(3,3,4)},$
 $(31, 44, 12, 26, 45, 43, 11, 27, 36)_{\Theta(3,3,4)},$
 $(25, 38, 36, 9, 33, 10, 1, 16, 37)_{\Theta(3,3,4)},$
 $(19, 39, 41, 46, 28, 47, 9, 52, 17)_{\Theta(3,3,4)},$
 $(37, 45, 11, 12, 34, 27, 40, 33, 46)_{\Theta(3,3,4)},$
 $(46, 6, 3, 12, 23, 51, 16, 33, 17)_{\Theta(3,3,4)},$
 $(16, 25, 12, 14, 54, 17, 19, \infty, 41)_{\Theta(3,3,4)},$
 $(2, 14, 19, 15, 50, 33, 34, 28, 48)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 5 \pmod{55}$, $\infty \mapsto \infty$.

K_{65} Let the vertex set be Z_{65} . The decompositions consist of the graphs

$(0, 4, 41, 14, 12, 22, 27, 7, 35)_{\Theta(1,2,7)},$
 $(55, 56, 52, 25, 27, 54, 34, 39, 26)_{\Theta(1,2,7)},$
 $(35, 59, 8, 13, 23, 55, 15, 14, 20)_{\Theta(1,2,7)},$
 $(42, 49, 60, 38, 12, 11, 20, 27, 62)_{\Theta(1,2,7)},$
 $(29, 63, 52, 54, 16, 18, 46, 31, 7)_{\Theta(1,2,7)},$
 $(17, 26, 6, 42, 14, 31, 38, 50, 29)_{\Theta(1,2,7)},$
 $(39, 6, 22, 47, 13, 49, 41, 46, 24)_{\Theta(1,2,7)},$
 $(28, 27, 3, 5, 21, 2, 8, 9, 13)_{\Theta(1,2,7)},$
 $(56, 31, 10, 8, 3, 52, 33, 11, 17)_{\Theta(1,2,7)},$
 $(22, 43, 56, 5, 63, 4, 57, 30, 40)_{\Theta(1,2,7)},$
 $(56, 17, 14, 48, 24, 33, 3, 60, 40)_{\Theta(1,2,7)},$
 $(33, 59, 44, 48, 29, 20, 39, 4, 6)_{\Theta(1,2,7)},$
 $(3, 55, 21, 23, 11, 4, 14, 50, 61)_{\Theta(1,2,7)},$
 $(50, 48, 21, 62, 26, 0, 28, 45, 30)_{\Theta(1,2,7)},$
 $(31, 21, 35, 63, 19, 3, 26, 47, 18)_{\Theta(1,2,7)},$
 $(0, 52, 60, 22, 7, 39, 17, 50, 34)_{\Theta(1,2,7)},$
 $(0, 17, 13, 46, 61, 52, 57, 6, 25)_{\Theta(1,3,6)},$
 $(29, 52, 24, 20, 15, 54, 16, 21, 50)_{\Theta(1,3,6)},$
 $(57, 61, 38, 19, 9, 11, 25, 48, 46)_{\Theta(1,3,6)},$
 $(49, 55, 48, 61, 17, 39, 54, 41, 2)_{\Theta(1,3,6)},$
 $(19, 12, 9, 3, 7, 6, 55, 23, 38)_{\Theta(1,3,6)},$
 $(33, 27, 60, 55, 8, 0, 26, 44, 62)_{\Theta(1,3,6)},$
 $(43, 51, 2, 29, 55, 40, 58, 31, 62)_{\Theta(1,3,6)},$
 $(4, 61, 18, 49, 15, 60, 30, 8, 44)_{\Theta(1,3,6)},$
 $(7, 9, 10, 54, 47, 60, 17, 62, 13)_{\Theta(1,3,6)},$
 $(15, 24, 42, 8, 25, 7, 17, 9, 37)_{\Theta(1,3,6)},$
 $(26, 33, 5, 12, 61, 13, 3, 44, 47)_{\Theta(1,3,6)},$
 $(42, 13, 57, 15, 21, 54, 29, 60, 20)_{\Theta(1,3,6)},$
 $(30, 21, 14, 3, 49, 28, 63, 26, 46)_{\Theta(1,3,6)},$
 $(15, 43, 26, 42, 44, 20, 3, 46, 23)_{\Theta(1,3,6)},$
 $(47, 58, 1, 63, 6, 9, 16, 28, 19)_{\Theta(1,3,6)},$
 $(0, 1, 3, 7, 31, 55, 54, 19, 56)_{\Theta(1,3,6)},$

$(0, 15, 23, 13, 31, 62, 42, 25, 61)_{\Theta(1,4,5)},$
 $(22, 54, 17, 14, 16, 36, 52, 58, 6)_{\Theta(1,4,5)},$
 $(57, 47, 32, 63, 17, 0, 51, 7, 29)_{\Theta(1,4,5)},$
 $(2, 28, 53, 40, 30, 44, 60, 20, 24)_{\Theta(1,4,5)},$
 $(11, 6, 17, 10, 51, 54, 20, 50, 62)_{\Theta(1,4,5)},$
 $(2, 40, 56, 48, 13, 23, 27, 26, 20)_{\Theta(1,4,5)},$
 $(20, 8, 15, 2, 0, 14, 46, 23, 51)_{\Theta(1,4,5)},$
 $(6, 10, 40, 58, 57, 37, 53, 14, 43)_{\Theta(1,4,5)},$
 $(29, 22, 42, 10, 49, 9, 58, 43, 0)_{\Theta(1,4,5)},$
 $(23, 14, 26, 33, 9, 30, 51, 50, 4)_{\Theta(1,4,5)},$
 $(24, 45, 36, 46, 4, 35, 44, 7, 48)_{\Theta(1,4,5)},$
 $(62, 14, 8, 28, 11, 53, 5, 16, 44)_{\Theta(1,4,5)},$
 $(15, 57, 29, 17, 21, 14, 58, 53, 59)_{\Theta(1,4,5)},$
 $(31, 40, 16, 18, 4, 1, 9, 34, 3)_{\Theta(1,4,5)},$
 $(56, 49, 32, 3, 36, 9, 28, 53, 64)_{\Theta(1,4,5)},$
 $(1, 13, 40, 12, 51, 26, 7, 22, 14)_{\Theta(1,4,5)},$
 $(0, 18, 23, 30, 52, 28, 43, 6, 1)_{\Theta(2,3,5)},$
 $(15, 60, 57, 42, 2, 34, 16, 9, 58)_{\Theta(2,3,5)},$
 $(15, 60, 36, 4, 29, 32, 50, 56, 24)_{\Theta(2,3,5)},$
 $(35, 54, 27, 36, 42, 34, 48, 12, 32)_{\Theta(2,3,5)},$
 $(0, 7, 46, 9, 54, 51, 37, 18, 11)_{\Theta(2,3,5)},$
 $(27, 26, 36, 15, 58, 60, 10, 43, 9)_{\Theta(2,3,5)},$
 $(9, 13, 57, 11, 34, 37, 22, 38, 27)_{\Theta(2,3,5)},$
 $(21, 29, 41, 52, 62, 2, 56, 34, 55)_{\Theta(2,3,5)},$
 $(2, 54, 46, 51, 16, 0, 48, 35, 43)_{\Theta(2,3,5)},$
 $(4, 42, 39, 20, 38, 33, 7, 31, 6)_{\Theta(2,3,5)},$
 $(24, 33, 9, 17, 23, 11, 58, 2, 30)_{\Theta(2,3,5)},$
 $(2, 27, 62, 44, 40, 59, 0, 5, 29)_{\Theta(2,3,5)},$
 $(30, 0, 56, 41, 25, 26, 54, 53, 14)_{\Theta(2,3,5)},$
 $(26, 48, 13, 28, 8, 29, 24, 37, 36)_{\Theta(2,3,5)},$
 $(56, 38, 41, 48, 29, 13, 12, 53, 11)_{\Theta(2,3,5)},$
 $(0, 63, 10, 36, 5, 45, 18, 14, 4)_{\Theta(2,3,5)},$
 $(0, 61, 29, 34, 36, 43, 27, 4, 62)_{\Theta(3,3,4)},$
 $(15, 56, 19, 1, 60, 39, 63, 53, 61)_{\Theta(3,3,4)},$
 $(7, 13, 52, 48, 33, 10, 41, 4, 56)_{\Theta(3,3,4)},$
 $(50, 7, 0, 35, 52, 53, 43, 1, 42)_{\Theta(3,3,4)},$
 $(13, 35, 42, 16, 45, 56, 14, 60, 61)_{\Theta(3,3,4)},$
 $(1, 34, 4, 7, 12, 35, 41, 57, 43)_{\Theta(3,3,4)},$
 $(24, 56, 1, 62, 52, 28, 42, 37, 49)_{\Theta(3,3,4)},$
 $(48, 61, 3, 16, 17, 4, 59, 29, 45)_{\Theta(3,3,4)},$
 $(4, 33, 15, 55, 37, 48, 24, 63, 58)_{\Theta(3,3,4)},$
 $(24, 1, 14, 5, 26, 60, 55, 12, 28)_{\Theta(3,3,4)},$
 $(21, 23, 53, 29, 35, 11, 2, 31, 19)_{\Theta(3,3,4)},$
 $(16, 26, 13, 61, 25, 17, 31, 52, 12)_{\Theta(3,3,4)},$
 $(36, 19, 38, 25, 4, 48, 14, 63, 45)_{\Theta(3,3,4)},$
 $(28, 43, 7, 29, 22, 35, 55, 4, 45)_{\Theta(3,3,4)},$
 $(2, 23, 57, 60, 20, 32, 50, 40, 35)_{\Theta(3,3,4)},$

$$(2, 29, 19, 4, 60, 27, 52, 44, 63)_{\Theta(3,3,4)}$$

under the action of the mapping $x \mapsto x + 5 \pmod{65}$. \square

Proof of Lemma 3.2

$K_{5,10}$ Let the vertex set be $\{0, 1, \dots, 14\}$ partitioned into $\{0, 3, 6, 9, 12\}$ and $\{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$. The decompositions consist of the graphs

$$(2, 13, 9, 0, 3, 8, 12, 7, 6)_{\Theta(2,2,6)}$$

under the action of the mapping $x \mapsto x + 3 \pmod{15}$, and

$$(5, 7, 0, 3, 4, 12, 9, 2, 6)_{\Theta(2,4,4)},$$

$$(1, 13, 6, 3, 11, 12, 9, 8, 0)_{\Theta(2,4,4)},$$

$$(11, 8, 6, 9, 7, 3, 0, 1, 12)_{\Theta(2,4,4)},$$

$$(4, 2, 0, 6, 10, 3, 9, 14, 12)_{\Theta(2,4,4)},$$

$$(10, 14, 0, 9, 13, 3, 12, 5, 6)_{\Theta(2,4,4)}.$$

$K_{10,10,10}$ Let the vertex set be Z_{30} partitioned according to residue class modulo 3. The decompositions consist of the graphs

$$(15, 28, 17, 1, 0, 4, 9, 16, 6)_{\Theta(1,2,7)},$$

$$(2, 3, 9, 1, 6, 11, 21, 5, 16)_{\Theta(1,3,6)},$$

$$(0, 17, 23, 1, 3, 4, 9, 8, 27)_{\Theta(1,4,5)},$$

$$(0, 1, 26, 17, 3, 7, 8, 19, 9)_{\Theta(2,3,5)},$$

$$(0, 19, 28, 15, 1, 8, 5, 13, 3)_{\Theta(3,3,4)}$$

under the action of the mapping $x \mapsto x + 1 \pmod{30}$.

$K_{5,5,5,5}$ Let the vertex set be $\{0, 1, \dots, 19\}$ partitioned into $\{3j + i : j = 0, 1, 2, 3, 4\}, i = 0, 1, 2$, and $\{15, 16, 17, 18, 19\}$. The decompositions consist of the graphs

$$(18, 0, 2, 1, 5, 4, 11, 6, 15)_{\Theta(1,2,7)},$$

$$(2, 18, 1, 3, 6, 11, 15, 8, 0)_{\Theta(1,3,6)},$$

$$(0, 17, 5, 13, 2, 14, 1, 15, 4)_{\Theta(1,4,5)},$$

$$(0, 18, 10, 14, 1, 4, 11, 16, 2)_{\Theta(2,3,5)},$$

$$(0, 16, 7, 8, 4, 2, 5, 15, 9)_{\Theta(3,3,4)}$$

under the action of the mapping $x \mapsto x + 1 \pmod{15}$ for $x < 15$, $x \mapsto (x + 1 \pmod{5}) + 15$ for $x \geq 15$.

$K_{20,20,20,25}$ Let the vertex set be $\{0, 1, \dots, 84\}$ partitioned into $\{3j + i : j = 0, 1, \dots, 19\}, i = 0, 1, 2$, and $\{60, 61, \dots, 84\}$. The decompositions consist of the graphs

$$(0, 63, 44, 69, 37, 12, 2, 33, 16)_{\Theta(1,2,7)},$$

$$(26, 72, 1, 64, 20, 80, 25, 44, 10)_{\Theta(1,2,7)},$$

$$(49, 29, 0, 3, 64, 27, 23, 36, 31)_{\Theta(1,2,7)},$$

$$(30, 61, 52, 60, 13, 47, 76, 34, 53)_{\Theta(1,2,7)},$$

$$(3, 26, 40, 62, 1, 45, 75, 7, 24)_{\Theta(1,2,7)},$$

$$(24, 61, 16, 31, 23, 83, 20, 25, 47)_{\Theta(1,2,7)},$$

$$(67, 56, 57, 0, 68, 43, 81, 7, 65)_{\Theta(1,2,7)},$$

$$(46, 59, 6, 80, 4, 8, 40, 62, 49)_{\Theta(1,2,7)},$$

$$(53, 78, 25, 74, 38, 37, 79, 8, 19)_{\Theta(1,2,7)},$$

$$(0, 10, 22, 66, 46, 74, 29, 78, 44)_{\Theta(1,3,6)},$$

$(72, 41, 25, 73, 47, 6, 78, 42, 69)_{\Theta(1,3,6)},$
 $(5, 43, 63, 54, 4, 15, 41, 57, 66)_{\Theta(1,3,6)},$
 $(14, 61, 10, 39, 60, 35, 62, 19, 47)_{\Theta(1,3,6)},$
 $(7, 54, 53, 10, 70, 5, 58, 80, 47)_{\Theta(1,3,6)},$
 $(34, 53, 33, 58, 54, 19, 15, 78, 48)_{\Theta(1,3,6)},$
 $(76, 39, 38, 69, 22, 59, 79, 57, 49)_{\Theta(1,3,6)},$
 $(25, 8, 38, 10, 45, 80, 24, 32, 74)_{\Theta(1,3,6)},$
 $(32, 72, 77, 16, 3, 5, 42, 60, 8)_{\Theta(1,3,6)},$
 $(0, 13, 60, 51, 75, 58, 83, 45, 64)_{\Theta(1,4,5)},$
 $(58, 71, 44, 78, 2, 17, 6, 28, 45)_{\Theta(1,4,5)},$
 $(73, 16, 15, 61, 50, 28, 32, 57, 64)_{\Theta(1,4,5)},$
 $(21, 34, 66, 48, 60, 65, 24, 67, 42)_{\Theta(1,4,5)},$
 $(61, 33, 52, 53, 37, 27, 17, 24, 34)_{\Theta(1,4,5)},$
 $(72, 45, 47, 79, 5, 33, 52, 35, 13)_{\Theta(1,4,5)},$
 $(46, 67, 62, 45, 40, 15, 49, 47, 39)_{\Theta(1,4,5)},$
 $(4, 24, 27, 70, 40, 41, 46, 18, 74)_{\Theta(1,4,5)},$
 $(15, 78, 29, 40, 11, 8, 79, 4, 39)_{\Theta(1,4,5)},$
 $(0, 23, 19, 52, 64, 60, 58, 72, 42)_{\Theta(2,3,5)},$
 $(65, 67, 1, 27, 22, 53, 24, 55, 15)_{\Theta(2,3,5)},$
 $(39, 9, 31, 68, 22, 65, 25, 50, 71)_{\Theta(2,3,5)},$
 $(56, 36, 19, 64, 50, 76, 53, 39, 29)_{\Theta(2,3,5)},$
 $(74, 32, 39, 41, 16, 43, 59, 57, 81)_{\Theta(2,3,5)},$
 $(40, 67, 39, 50, 27, 81, 1, 12, 13)_{\Theta(2,3,5)},$
 $(80, 69, 30, 44, 6, 2, 78, 21, 53)_{\Theta(2,3,5)},$
 $(45, 44, 68, 71, 12, 28, 41, 63, 24)_{\Theta(2,3,5)},$
 $(57, 50, 46, 83, 16, 23, 28, 26, 72)_{\Theta(2,3,5)},$
 $(0, 48, 5, 61, 14, 80, 75, 55, 26)_{\Theta(3,3,4)},$
 $(39, 51, 81, 50, 56, 68, 44, 79, 37)_{\Theta(3,3,4)},$
 $(3, 51, 47, 62, 23, 74, 14, 24, 83)_{\Theta(3,3,4)},$
 $(16, 11, 33, 60, 69, 45, 57, 32, 48)_{\Theta(3,3,4)},$
 $(45, 63, 62, 18, 49, 30, 14, 1, 2)_{\Theta(3,3,4)},$
 $(34, 23, 54, 71, 32, 76, 82, 2, 15)_{\Theta(3,3,4)},$
 $(78, 29, 21, 19, 23, 51, 17, 65, 54)_{\Theta(3,3,4)},$
 $(52, 38, 81, 31, 20, 82, 59, 77, 15)_{\Theta(3,3,4)},$
 $(26, 41, 52, 70, 64, 30, 34, 38, 79)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 2 \pmod{60}$ for $x < 60$, $x \mapsto (x - 60 + 5 \pmod{25}) + 60$ for $x \geq 60$.

$K_{5,5,5,5,5}$ Let the vertex set be Z_{25} partitioned according to residue class modulo 5. The decompositions consist of the graphs

$(4, 8, 21, 1, 0, 2, 9, 3, 17)_{\Theta(1,2,7)},$
 $(3, 19, 0, 1, 5, 9, 20, 8, 2)_{\Theta(1,3,6)},$
 $(0, 19, 13, 5, 1, 9, 6, 4, 18)_{\Theta(1,4,5)},$
 $(0, 22, 21, 2, 5, 9, 3, 15, 4)_{\Theta(2,3,5)},$
 $(0, 22, 2, 21, 9, 1, 3, 15, 4)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 1 \pmod{25}$.

$K_{5,5,5,5,15}$ Let the vertex set be $\{0, 1, \dots, 34\}$ partitioned into $\{3j + i : j = 0, 1, 2, 3, 4\}$, $i = 0, 1, 2$, $\{15, 16, 17, 18, 19\}$ and $\{20, 21, \dots, 34\}$. The decompositions consist of the graphs

$$\begin{aligned}
& (5, 26, 6, 15, 22, 16, 4, 8, 1)_{\Theta(1,2,7)}, \\
& (21, 17, 3, 13, 28, 1, 32, 8, 30)_{\Theta(1,2,7)}, \\
& (15, 14, 12, 20, 1, 6, 32, 0, 28)_{\Theta(1,2,7)}, \\
& (4, 20, 23, 12, 6, 24, 3, 11, 10)_{\Theta(1,3,6)}, \\
& (7, 21, 16, 1, 24, 12, 25, 18, 6)_{\Theta(1,3,6)}, \\
& (14, 10, 15, 34, 4, 26, 16, 24, 18)_{\Theta(1,3,6)}, \\
& (6, 24, 13, 23, 9, 16, 33, 14, 18)_{\Theta(1,4,5)}, \\
& (7, 5, 33, 19, 6, 11, 17, 32, 0)_{\Theta(1,4,5)}, \\
& (6, 27, 18, 21, 5, 30, 7, 20, 0)_{\Theta(1,4,5)}, \\
& (23, 8, 13, 18, 20, 3, 29, 1, 9)_{\Theta(2,3,5)}, \\
& (10, 9, 15, 17, 25, 8, 29, 7, 26)_{\Theta(2,3,5)}, \\
& (33, 1, 19, 0, 24, 3, 7, 21, 15)_{\Theta(2,3,5)}, \\
& (8, 32, 31, 15, 25, 3, 29, 16, 13)_{\Theta(3,3,4)}, \\
& (27, 28, 14, 4, 0, 13, 1, 26, 10)_{\Theta(3,3,4)}, \\
& (27, 6, 7, 16, 17, 5, 18, 2, 10)_{\Theta(3,3,4)}
\end{aligned}$$

under the action of the mapping $x \mapsto x + 1 \pmod{15}$ for $x < 15$, $x \mapsto (x + 1 \pmod{5}) + 15$ for $15 \leq x < 20$, $x \mapsto (x - 20 + 1 \pmod{15}) + 20$ for $x \geq 20$.

$K_{5,5,5,5,20}$ Let the vertex set be $\{0, 1, \dots, 39\}$ partitioned into $\{3j + i : j = 0, 1, 2, 3, 4\}$, $i = 0, 1, 2, 3$ and $\{20, 21, \dots, 39\}$. The decompositions consist of the graphs

$$\begin{aligned}
& (16, 37, 17, 34, 4, 20, 15, 26, 1)_{\Theta(1,2,7)}, \\
& (27, 10, 4, 6, 21, 16, 7, 13, 28)_{\Theta(1,2,7)}, \\
& (2, 34, 12, 15, 28, 1, 6, 9, 0)_{\Theta(1,2,7)}, \\
& (30, 1, 3, 14, 20, 19, 31, 13, 34)_{\Theta(1,2,7)}, \\
& (12, 27, 11, 38, 19, 2, 32, 18, 16)_{\Theta(1,2,7)}, \\
& (35, 8, 1, 10, 30, 6, 11, 4, 32)_{\Theta(1,2,7)}, \\
& (0, 33, 14, 29, 6, 35, 7, 27, 3)_{\Theta(1,2,7)}, \\
& (14, 16, 1, 22, 5, 28, 9, 21, 19)_{\Theta(1,2,7)}, \\
& (3, 37, 13, 38, 15, 32, 12, 29, 11)_{\Theta(1,2,7)}, \\
& (31, 9, 18, 1, 19, 4, 36, 14, 15)_{\Theta(1,2,7)}, \\
& (1, 29, 2, 27, 8, 5, 36, 7, 18)_{\Theta(1,2,7)}, \\
& (14, 25, 24, 0, 35, 2, 26, 9, 7)_{\Theta(1,3,6)}, \\
& (17, 38, 0, 3, 24, 15, 20, 5, 10)_{\Theta(1,3,6)}, \\
& (37, 9, 4, 32, 8, 29, 10, 17, 11)_{\Theta(1,3,6)}, \\
& (8, 17, 28, 6, 14, 19, 36, 0, 23)_{\Theta(1,3,6)}, \\
& (12, 23, 24, 11, 13, 33, 6, 21, 5)_{\Theta(1,3,6)}, \\
& (33, 11, 1, 31, 19, 12, 10, 30, 5)_{\Theta(1,3,6)}, \\
& (4, 15, 38, 0, 30, 11, 37, 14, 31)_{\Theta(1,3,6)}, \\
& (21, 4, 17, 31, 11, 35, 13, 26, 3)_{\Theta(1,3,6)}, \\
& (12, 6, 14, 3, 27, 8, 13, 0, 22)_{\Theta(1,3,6)}, \\
& (27, 18, 2, 36, 19, 6, 7, 38, 8)_{\Theta(1,3,6)}, \\
& (3, 30, 24, 18, 13, 10, 9, 20, 1)_{\Theta(1,3,6)}, \\
& (8, 11, 19, 33, 9, 15, 22, 0, 32)_{\Theta(1,4,5)}, \\
& (16, 29, 36, 12, 7, 2, 15, 10, 0)_{\Theta(1,4,5)},
\end{aligned}$$

$(37, 6, 0, 23, 8, 18, 1, 19, 38)_{\Theta(1,4,5)},$
 $(13, 20, 22, 8, 17, 19, 28, 2, 3)_{\Theta(1,4,5)},$
 $(27, 7, 15, 38, 13, 5, 35, 2, 8)_{\Theta(1,4,5)},$
 $(28, 17, 15, 21, 18, 3, 31, 5, 25)_{\Theta(1,4,5)},$
 $(18, 20, 33, 15, 6, 23, 14, 21, 5)_{\Theta(1,4,5)},$
 $(18, 35, 36, 0, 1, 38, 5, 14, 11)_{\Theta(1,4,5)},$
 $(36, 17, 8, 33, 12, 6, 1, 14, 35)_{\Theta(1,4,5)},$
 $(12, 38, 23, 7, 17, 31, 4, 22, 14)_{\Theta(1,4,5)},$
 $(3, 38, 33, 1, 8, 34, 17, 0, 2)_{\Theta(1,4,5)},$
 $(8, 13, 35, 18, 12, 23, 9, 33, 19)_{\Theta(2,3,5)},$
 $(11, 16, 29, 33, 3, 1, 30, 7, 35)_{\Theta(2,3,5)},$
 $(13, 31, 11, 29, 8, 2, 5, 24, 14)_{\Theta(2,3,5)},$
 $(23, 2, 19, 11, 22, 10, 36, 15, 30)_{\Theta(2,3,5)},$
 $(35, 37, 10, 19, 0, 14, 28, 16, 18)_{\Theta(2,3,5)},$
 $(8, 33, 2, 19, 5, 1, 3, 36, 18)_{\Theta(2,3,5)},$
 $(16, 21, 1, 38, 12, 24, 9, 2, 15)_{\Theta(2,3,5)},$
 $(30, 31, 13, 12, 1, 17, 38, 8, 5)_{\Theta(2,3,5)},$
 $(29, 28, 17, 4, 19, 6, 38, 11, 8)_{\Theta(2,3,5)},$
 $(36, 34, 9, 0, 18, 11, 10, 32, 15)_{\Theta(2,3,5)},$
 $(0, 1, 24, 31, 2, 34, 10, 15, 6)_{\Theta(2,3,5)},$
 $(29, 35, 1, 0, 19, 12, 18, 13, 16)_{\Theta(3,3,4)},$
 $(13, 4, 3, 38, 35, 10, 33, 16, 7)_{\Theta(3,3,4)},$
 $(36, 25, 2, 0, 13, 6, 8, 30, 9)_{\Theta(3,3,4)},$
 $(10, 5, 19, 32, 3, 34, 20, 9, 0)_{\Theta(3,3,4)},$
 $(20, 6, 8, 7, 3, 28, 4, 17, 11)_{\Theta(3,3,4)},$
 $(18, 2, 1, 25, 4, 37, 31, 10, 12)_{\Theta(3,3,4)},$
 $(4, 31, 33, 7, 22, 14, 28, 9, 15)_{\Theta(3,3,4)},$
 $(11, 27, 25, 13, 29, 7, 16, 36, 1)_{\Theta(3,3,4)},$
 $(4, 34, 30, 11, 25, 18, 35, 7, 9)_{\Theta(3,3,4)},$
 $(13, 22, 26, 2, 14, 5, 15, 36, 10)_{\Theta(3,3,4)},$
 $(2, 23, 20, 11, 31, 13, 19, 26, 16)_{\Theta(3,3,4)}$

under the action of the mapping $x \mapsto x + 4 \pmod{20}$ for $x < 20$, $x \mapsto (x + 4 \pmod{20}) + 20$ for $x \geq 20$. \square

B Theta graphs of 11 edges

Proof of Lemma 3.3

K_{11} Let the vertex set be $Z_{10} \cup \{\infty\}$. The decompositions consist of the graphs

$(0, 2, 1, 3, 7, 5, 8, 4, 9, \infty)_{\Theta(1,2,8)},$
 $(0, 2, 1, 5, 4, 3, 6, \infty, 9, 7)_{\Theta(1,3,7)},$
 $(0, 2, 1, 3, 6, 9, 5, \infty, 4, 7)_{\Theta(1,4,6)},$
 $(0, 2, 1, 3, 4, 7, 6, \infty, 5, 9)_{\Theta(1,5,5)},$
 $(0, 2, 1, 4, 3, 6, \infty, 5, 9, 7)_{\Theta(2,2,7)},$
 $(0, 2, 1, 3, 6, 5, 9, 7, \infty, 4)_{\Theta(2,3,6)},$

$$(0, 2, 1, 3, 5, 9, 4, 6, \infty, 7)_{\Theta(2,4,5)},$$

$$(0, 2, 1, 4, 3, 8, 9, 5, 7, \infty)_{\Theta(3,3,5)},$$

$$(0, 2, 1, 4, 3, 9, 7, 6, 5, \infty)_{\Theta(3,4,4)}$$

under the action of the mapping $x \mapsto x + 2 \pmod{10}$, $\infty \mapsto \infty$.

K_{12} Let the vertex set be Z_{12} . The decompositions consist of

$$(0, 1, 2, 3, 4, 8, 5, 7, 6, 9)_{\Theta(1,2,8)},$$

$$(2, 7, 9, 4, 10, 6, 3, 11, 5, 0)_{\Theta(1,2,8)},$$

$$(0, 1, 2, 3, 4, 7, 8, 5, 6, 9)_{\Theta(1,3,7)},$$

$$(2, 7, 4, 9, 8, 3, 6, 10, 5, 11)_{\Theta(1,3,7)},$$

$$(0, 1, 2, 3, 4, 5, 6, 8, 11, 7)_{\Theta(1,4,6)},$$

$$(2, 7, 6, 3, 5, 9, 1, 10, 4, 0)_{\Theta(1,4,6)},$$

$$(0, 1, 2, 3, 4, 7, 5, 6, 9, 11)_{\Theta(1,5,5)},$$

$$(2, 7, 4, 0, 6, 10, 9, 5, 8, 3)_{\Theta(1,5,5)},$$

$$(0, 1, 2, 3, 4, 5, 8, 6, 7, 9)_{\Theta(2,2,7)},$$

$$(2, 3, 6, 7, 8, 1, 10, 5, 11, 4)_{\Theta(2,2,7)},$$

$$(0, 1, 2, 3, 4, 5, 7, 6, 8, 9)_{\Theta(2,3,6)},$$

$$(2, 3, 6, 5, 10, 8, 0, 7, 1, 11)_{\Theta(2,3,6)},$$

$$(0, 1, 2, 3, 4, 5, 6, 9, 11, 7)_{\Theta(2,4,5)},$$

$$(2, 3, 10, 9, 7, 6, 4, 0, 5, 8)_{\Theta(2,4,5)},$$

$$(0, 1, 2, 3, 4, 5, 6, 8, 7, 10)_{\Theta(3,3,5)},$$

$$(2, 7, 1, 4, 6, 11, 9, 3, 5, 0)_{\Theta(3,3,5)},$$

$$(0, 1, 2, 3, 4, 5, 6, 7, 11, 8)_{\Theta(3,4,4)},$$

$$(2, 3, 5, 9, 6, 0, 10, 11, 1, 4)_{\Theta(3,4,4)}$$

under the action of the mapping $x \mapsto x + 4 \pmod{12}$.

K_{22} Let the vertex set of be $Z_{21} \cup \{\infty\}$. The decompositions consist of

$$(\infty, 0, 1, 2, 3, 5, 4, 6, 9, 13)_{\Theta(1,2,8)},$$

$$(0, 5, 7, 6, 1, 4, 8, 2, 9, 17)_{\Theta(1,2,8)},$$

$$(0, 10, 17, 9, 20, 2, 7, 1, 11, 19)_{\Theta(1,2,8)},$$

$$(\infty, 0, 1, 2, 3, 5, 7, 4, 6, 8)_{\Theta(1,4,6)},$$

$$(0, 1, 4, 8, 11, 5, 9, 2, 7, 12)_{\Theta(1,4,6)},$$

$$(0, 11, 6, 13, 4, 9, 1, 16, 8, 2)_{\Theta(1,4,6)},$$

$$(\infty, 0, 1, 2, 3, 6, 4, 5, 7, 10)_{\Theta(2,2,7)},$$

$$(0, 1, 5, 6, 4, 10, 2, 3, 11, 8)_{\Theta(2,2,7)},$$

$$(0, 2, 7, 9, 11, 17, 8, 12, 4, 13)_{\Theta(2,2,7)},$$

$$(\infty, 0, 1, 2, 3, 6, 4, 5, 7, 10)_{\Theta(2,3,6)},$$

$$(0, 1, 5, 2, 6, 4, 10, 3, 9, 17)_{\Theta(2,3,6)},$$

$$(0, 2, 9, 11, 5, 13, 1, 8, 20, 10)_{\Theta(2,3,6)},$$

$$(\infty, 0, 1, 2, 3, 5, 6, 4, 7, 8)_{\Theta(2,4,5)},$$

$$(0, 1, 6, 3, 7, 5, 9, 2, 8, 11)_{\Theta(2,4,5)},$$

$$(0, 2, 7, 10, 1, 14, 11, 4, 19, 6)_{\Theta(2,4,5)},$$

$$(\infty, 0, 1, 2, 3, 4, 6, 5, 7, 10)_{\Theta(3,4,4)},$$

$$(0, 1, 3, 7, 5, 2, 6, 8, 4, 11)_{\Theta(3,4,4)},$$

$$(0, 2, 9, 8, 7, 16, 11, 13, 5, 12)_{\Theta(3,4,4)}$$

under the action of the mapping $x \mapsto x + 3 \pmod{21}$, $\infty \mapsto \infty$. \square

Proof of Lemma 3.4

$K_{11,11}$ Let the vertex set be Z_{22} partitioned according to residue classes modulo 2. The decompositions consist of

$$(0, 1, 3, 4, 5, 16, 7, 20, 13, 6)_{\Theta(1,3,7)},$$

$$(0, 1, 3, 4, 9, 12, 15, 2, 11, 16)_{\Theta(1,5,5)},$$

$$(0, 1, 3, 2, 5, 8, 7, 12, 21, 10)_{\Theta(3,3,5)}$$

under the action of the mapping $x \mapsto x + 2 \pmod{22}$.

$K_{11,11,11}$ Let the vertex set be Z_{33} partitioned according to residue classes modulo 3. The decompositions consist of

$$(0, 1, 5, 2, 9, 17, 3, 14, 4, 21)_{\Theta(1,2,8)},$$

$$(0, 1, 2, 6, 11, 8, 19, 5, 21, 14)_{\Theta(1,4,6)},$$

$$(0, 1, 2, 5, 7, 15, 26, 10, 24, 11)_{\Theta(2,2,7)},$$

$$(0, 1, 2, 4, 9, 7, 20, 3, 22, 11)_{\Theta(2,3,6)},$$

$$(0, 1, 2, 4, 9, 17, 7, 18, 5, 15)_{\Theta(2,4,5)},$$

$$(0, 1, 2, 6, 7, 15, 14, 10, 26, 12)_{\Theta(3,4,4)}$$

under the action of the mapping $x \mapsto x + 1 \pmod{33}$.

$K_{11,11,11,11}$ Let the vertex set be $\{0, 1, \dots, 43\}$ partitioned into $\{3j + i : j = 0, 1, \dots, 10\}$, $i = 0, 1, 2$, and $\{33, 34, \dots, 43\}$. The decompositions consist of

$$(0, 10, 32, 37, 8, 38, 14, 31, 24, 35)_{\Theta(1,2,8)},$$

$$(5, 37, 3, 0, 4, 12, 26, 13, 40, 9)_{\Theta(1,2,8)},$$

$$(0, 33, 13, 30, 29, 28, 35, 19, 27, 16)_{\Theta(1,4,6)},$$

$$(27, 17, 1, 3, 37, 8, 4, 34, 9, 41)_{\Theta(1,4,6)},$$

$$(0, 27, 20, 17, 2, 36, 4, 32, 34, 5)_{\Theta(2,2,7)},$$

$$(17, 35, 9, 13, 3, 4, 34, 6, 37, 10)_{\Theta(2,2,7)},$$

$$(0, 31, 17, 26, 27, 22, 24, 33, 4, 39)_{\Theta(2,3,6)},$$

$$(33, 6, 1, 5, 42, 0, 20, 10, 2, 40)_{\Theta(2,3,6)},$$

$$(0, 12, 42, 26, 28, 32, 33, 9, 34, 2)_{\Theta(2,4,5)},$$

$$(40, 3, 11, 1, 6, 17, 2, 18, 41, 4)_{\Theta(2,4,5)},$$

$$(0, 21, 38, 29, 33, 32, 1, 40, 5, 4)_{\Theta(3,4,4)},$$

$$(38, 8, 1, 12, 2, 21, 40, 10, 3, 13)_{\Theta(3,4,4)}$$

under the action of the mapping $x \mapsto x + 1 \pmod{33}$ for $x < 33$, $x \mapsto (x + 1 \pmod{11}) + 33$ for $x \geq 33$.

$K_{11,11,11,11,11}$ Let the vertex set be Z_{55} partitioned according to residue class modulo 5. The decompositions consist of

$$(0, 46, 2, 26, 38, 1, 20, 21, 37, 15)_{\Theta(1,2,8)},$$

$$(7, 4, 0, 1, 9, 37, 14, 48, 31, 17)_{\Theta(1,2,8)},$$

$$(0, 38, 41, 47, 20, 31, 15, 26, 14, 12)_{\Theta(1,4,6)},$$

$$(34, 27, 0, 1, 4, 12, 16, 24, 5, 14)_{\Theta(1,4,6)},$$

$(0, 21, 32, 18, 4, 51, 44, 28, 30, 43)_{\Theta(2,2,7)},$
 $(34, 3, 15, 17, 5, 6, 12, 21, 0, 27)_{\Theta(2,2,7)},$
 $(0, 42, 38, 41, 19, 3, 1, 10, 21, 48)_{\Theta(2,3,6)},$
 $(6, 21, 5, 13, 0, 24, 12, 4, 23, 47)_{\Theta(2,3,6)},$
 $(0, 2, 16, 22, 45, 46, 27, 25, 28, 40)_{\Theta(2,4,5)},$
 $(20, 26, 2, 1, 9, 5, 7, 0, 6, 35)_{\Theta(2,4,5)},$
 $(0, 26, 46, 34, 39, 37, 50, 27, 5, 12)_{\Theta(3,4,4)},$
 $(47, 34, 3, 2, 10, 4, 0, 18, 1, 37)_{\Theta(3,4,4)}$

under the action of the mapping $x \mapsto x + 1 \pmod{55}$. □

C Theta graphs of 12 edges

Proof of Lemma 3.5

K_{16} Let the vertex set be $Z_{15} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 0, 1, 2, 3, 5, 4, 6, 9, 13, 7)_{\Theta(1,2,9)},$
 $(1, 5, 8, 4, 2, 7, 12, 3, 14, 6, 11)_{\Theta(1,2,9)},$
 $(\infty, 0, 1, 2, 5, 6, 3, 4, 7, 11, 8)_{\Theta(1,3,8)},$
 $(1, 8, 3, 10, 6, 0, 5, 9, 13, 4, 14)_{\Theta(1,3,8)},$
 $(\infty, 0, 1, 2, 3, 5, 7, 4, 6, 8, 11)_{\Theta(1,4,7)},$
 $(1, 5, 0, 4, 10, 9, 3, 8, 14, 7, 12)_{\Theta(1,4,7)},$
 $(\infty, 0, 1, 2, 3, 4, 5, 7, 9, 6, 8)_{\Theta(1,5,6)},$
 $(1, 5, 4, 9, 0, 11, 7, 2, 10, 3, 8)_{\Theta(1,5,6)},$
 $(\infty, 0, 1, 2, 3, 6, 4, 5, 7, 10, 14)_{\Theta(2,2,8)},$
 $(1, 2, 6, 7, 8, 5, 0, 4, 12, 3, 11)_{\Theta(2,2,8)},$
 $(\infty, 0, 1, 2, 3, 6, 4, 5, 7, 11, 8)_{\Theta(2,3,7)},$
 $(1, 2, 6, 4, 0, 7, 14, 5, 10, 3, 12)_{\Theta(2,3,7)},$
 $(\infty, 0, 1, 2, 3, 5, 6, 4, 7, 8, 10)_{\Theta(2,4,6)},$
 $(1, 2, 5, 8, 0, 11, 12, 3, 6, 13, 7)_{\Theta(2,4,6)},$
 $(\infty, 0, 1, 2, 3, 5, 4, 6, 9, 7, 10)_{\Theta(2,5,5)},$
 $(1, 2, 5, 8, 0, 6, 11, 10, 3, 14, 4)_{\Theta(2,5,5)},$
 $(\infty, 0, 1, 2, 3, 4, 5, 6, 9, 7, 10)_{\Theta(3,3,6)},$
 $(1, 2, 5, 7, 8, 11, 10, 3, 9, 14, 6)_{\Theta(3,3,6)},$
 $(\infty, 0, 1, 2, 3, 4, 6, 5, 7, 10, 14)_{\Theta(3,4,5)},$
 $(1, 2, 6, 9, 7, 3, 10, 11, 0, 5, 8)_{\Theta(3,4,5)},$
 $(\infty, 0, 1, 2, 3, 5, 7, 4, 6, 8, 11)_{\Theta(4,4,4)},$
 $(1, 2, 0, 5, 10, 6, 4, 13, 9, 3, 11)_{\Theta(4,4,4)}$

under the action of the mapping $x \mapsto x + 3 \pmod{15}$, $\infty \mapsto \infty$.

K_{33} Let the vertex set be Z_{33} . The decompositions consist of

$(0, 4, 31, 10, 18, 27, 9, 11, 19, 29, 20)_{\Theta(1,2,9)},$
 $(5, 24, 17, 0, 13, 14, 1, 27, 28, 26, 11)_{\Theta(1,2,9)},$
 $(28, 25, 4, 2, 13, 30, 18, 14, 8, 5, 6)_{\Theta(1,2,9)},$
 $(7, 2, 12, 11, 0, 6, 3, 20, 1, 16, 27)_{\Theta(1,2,9)},$

$(0, 3, 19, 25, 6, 1, 2, 16, 29, 21, 4)_{\Theta(1,3,8)},$
 $(11, 18, 5, 14, 19, 6, 16, 7, 10, 28, 3)_{\Theta(1,3,8)},$
 $(1, 30, 23, 20, 5, 10, 26, 6, 17, 2, 21)_{\Theta(1,3,8)},$
 $(10, 8, 3, 1, 22, 32, 11, 6, 23, 21, 9)_{\Theta(1,3,8)},$
 $(0, 18, 30, 26, 10, 24, 1, 3, 17, 19, 13)_{\Theta(1,4,7)},$
 $(20, 10, 2, 27, 14, 3, 19, 12, 23, 29, 1)_{\Theta(1,4,7)},$
 $(9, 11, 8, 20, 19, 22, 10, 13, 27, 16, 2)_{\Theta(1,4,7)},$
 $(10, 6, 9, 2, 12, 17, 28, 13, 26, 23, 18)_{\Theta(1,4,7)},$
 $(0, 11, 24, 1, 6, 5, 4, 3, 28, 15, 18)_{\Theta(1,5,6)},$
 $(19, 22, 13, 15, 20, 2, 4, 5, 28, 6, 29)_{\Theta(1,5,6)},$
 $(20, 23, 16, 5, 29, 4, 3, 30, 13, 8, 27)_{\Theta(1,5,6)},$
 $(3, 5, 10, 19, 0, 18, 15, 23, 7, 28, 26)_{\Theta(1,5,6)},$
 $(0, 1, 23, 21, 29, 27, 3, 10, 28, 18, 15)_{\Theta(2,2,8)},$
 $(25, 14, 28, 27, 31, 29, 30, 8, 26, 0, 4)_{\Theta(2,2,8)},$
 $(17, 12, 1, 4, 16, 8, 3, 20, 25, 30, 13)_{\Theta(2,2,8)},$
 $(8, 2, 11, 14, 0, 18, 12, 26, 19, 7, 31)_{\Theta(2,2,8)},$
 $(0, 15, 2, 30, 23, 28, 5, 26, 10, 19, 21)_{\Theta(2,3,7)},$
 $(1, 0, 5, 7, 29, 8, 17, 27, 10, 22, 15)_{\Theta(2,3,7)},$
 $(0, 13, 9, 19, 5, 12, 23, 25, 20, 3, 16)_{\Theta(2,3,7)},$
 $(2, 5, 8, 1, 24, 17, 4, 12, 13, 28, 6)_{\Theta(2,3,7)},$
 $(0, 5, 27, 4, 2, 18, 14, 9, 16, 6, 29)_{\Theta(2,4,6)},$
 $(20, 5, 17, 10, 22, 16, 25, 11, 7, 15, 31)_{\Theta(2,4,6)},$
 $(18, 1, 21, 19, 0, 2, 11, 3, 15, 30, 26)_{\Theta(2,4,6)},$
 $(28, 2, 8, 4, 1, 17, 0, 9, 7, 25, 3)_{\Theta(2,4,6)},$
 $(0, 16, 17, 19, 29, 13, 6, 15, 10, 1, 23)_{\Theta(2,5,5)},$
 $(28, 11, 30, 15, 19, 31, 25, 8, 4, 1, 9)_{\Theta(2,5,5)},$
 $(3, 17, 6, 24, 15, 5, 12, 9, 29, 4, 2)_{\Theta(2,5,5)},$
 $(14, 2, 5, 6, 7, 24, 23, 19, 1, 12, 8)_{\Theta(2,5,5)},$
 $(0, 18, 10, 31, 23, 3, 27, 15, 17, 26, 1)_{\Theta(3,3,6)},$
 $(19, 8, 15, 13, 0, 7, 5, 17, 2, 25, 24)_{\Theta(3,3,6)},$
 $(4, 17, 22, 19, 11, 6, 26, 29, 30, 27, 23)_{\Theta(3,3,6)},$
 $(21, 0, 2, 9, 13, 22, 29, 25, 31, 11, 28)_{\Theta(3,3,6)},$
 $(0, 18, 29, 11, 15, 26, 24, 19, 8, 20, 10)_{\Theta(3,4,5)},$
 $(8, 14, 4, 6, 5, 18, 30, 2, 3, 19, 1)_{\Theta(3,4,5)},$
 $(6, 31, 28, 24, 11, 21, 18, 1, 4, 5, 14)_{\Theta(3,4,5)},$
 $(24, 1, 0, 10, 5, 7, 13, 25, 11, 4, 29)_{\Theta(3,4,5)},$
 $(0, 20, 26, 27, 13, 31, 7, 4, 18, 12, 2)_{\Theta(4,4,4)},$
 $(17, 5, 3, 31, 8, 9, 25, 13, 28, 1, 21)_{\Theta(4,4,4)},$
 $(28, 30, 24, 15, 17, 3, 14, 18, 29, 31, 2)_{\Theta(4,4,4)},$
 $(20, 1, 7, 30, 0, 11, 16, 27, 26, 5, 19)_{\Theta(4,4,4)}$

under the action of the mapping $x \mapsto x + 3 \pmod{33}$.

K_{40} Let the vertex set be $Z_{39} \cup \{\infty\}$. The decompositions consist of

$(\infty, 31, 0, 2, 27, 17, 16, 19, 37, 11, 26)_{\Theta(1,2,9)},$
 $(2, 4, 19, 29, 20, 38, 36, 25, 30, 8, 16)_{\Theta(1,2,9)},$
 $(33, 20, 0, 12, 24, 29, 23, 19, 15, 31, 9)_{\Theta(1,2,9)},$

$(4, 30, 29, 20, 10, 38, 19, 28, 3, 18, 21)_{\Theta(1,2,9)},$
 $(21, 14, 17, 5, 36, 4, 24, 22, 12, 13, 7)_{\Theta(1,2,9)},$
 $(\infty, 26, 31, 12, 24, 2, 22, 33, 9, 29, 28)_{\Theta(1,3,8)},$
 $(11, 23, 14, 21, 0, 36, 1, 28, 4, 35, 17)_{\Theta(1,3,8)},$
 $(13, 27, 16, 25, 8, 0, 6, 7, 30, 22, 1)_{\Theta(1,3,8)},$
 $(17, 21, 28, 5, 12, 22, 15, 27, 14, 38, 0)_{\Theta(1,3,8)},$
 $(14, 24, 1, 7, 4, 8, 22, 5, 35, 28, 33)_{\Theta(1,3,8)},$
 $(\infty, 20, 25, 37, 31, 6, 28, 9, 5, 26, 17)_{\Theta(1,4,7)},$
 $(0, 23, 16, 11, 1, 29, 37, 27, 28, 36, 22)_{\Theta(1,4,7)},$
 $(35, 15, 16, 23, 10, 27, 12, 14, 28, 30, 33)_{\Theta(1,4,7)},$
 $(32, 16, 26, 11, 7, 34, 10, 13, 24, 23, 12)_{\Theta(1,4,7)},$
 $(5, 0, 12, 3, 17, 32, 6, 13, 34, 21, 27)_{\Theta(1,4,7)},$
 $(\infty, 2, 18, 31, 20, 19, 13, 25, 6, 32, 33)_{\Theta(1,5,6)},$
 $(26, 5, 21, 4, 11, 38, 24, 1, 34, 8, 10)_{\Theta(1,5,6)},$
 $(9, 23, 18, 12, 8, 4, 16, 19, 37, 13, 38)_{\Theta(1,5,6)},$
 $(23, 33, 20, 28, 0, 25, 3, 6, 1, 30, 34)_{\Theta(1,5,6)},$
 $(9, 7, 2, 11, 0, 17, 24, 3, 30, 14, 37)_{\Theta(1,5,6)},$
 $(\infty, 29, 19, 11, 7, 0, 21, 18, 13, 8, 12)_{\Theta(2,3,7)},$
 $(37, 23, 5, 28, 10, 4, 30, 13, 11, 9, 18)_{\Theta(2,3,7)},$
 $(15, 29, 23, 22, 37, 30, 3, 31, 19, 35, 36)_{\Theta(2,3,7)},$
 $(27, 10, 33, 2, 38, 25, 0, 29, 14, 34, 35)_{\Theta(2,3,7)},$
 $(0, 1, 4, 10, 9, 23, 3, 29, 2, 32, 21)_{\Theta(2,3,7)},$
 $(\infty, 1, 10, 24, 19, 20, 14, 32, 11, 36, 15)_{\Theta(2,5,5)},$
 $(13, 32, 33, 37, 16, 28, 30, 17, 25, 11, 15)_{\Theta(2,5,5)},$
 $(29, 15, 7, 22, 11, 18, 24, 31, 8, 3, 19)_{\Theta(2,5,5)},$
 $(29, 12, 19, 14, 26, 23, 15, 3, 25, 24, 0)_{\Theta(2,5,5)},$
 $(0, 1, 20, 10, 5, 15, 4, 23, 32, 21, 34)_{\Theta(2,5,5)},$
 $(\infty, 14, 17, 3, 4, 12, 6, 23, 31, 7, 26)_{\Theta(3,3,6)},$
 $(37, 38, 35, 13, 7, 10, 4, 36, 32, 25, 0)_{\Theta(3,3,6)},$
 $(28, 27, 1, 14, 9, 0, 38, 5, 8, 4, 21)_{\Theta(3,3,6)},$
 $(13, 18, 9, 2, 8, 38, 36, 10, 21, 31, 33)_{\Theta(3,3,6)},$
 $(15, 2, 5, 20, 12, 33, 10, 11, 6, 7, 25)_{\Theta(3,3,6)},$
 $(\infty, 30, 5, 1, 33, 38, 19, 28, 9, 17, 14)_{\Theta(3,4,5)},$
 $(36, 38, 15, 22, 13, 1, 29, 24, 0, 3, 25)_{\Theta(3,4,5)},$
 $(31, 35, 32, 6, 22, 36, 8, 38, 17, 0, 33)_{\Theta(3,4,5)},$
 $(38, 12, 14, 8, 28, 27, 26, 16, 18, 22, 7)_{\Theta(3,4,5)},$
 $(29, 2, 4, 7, 31, 18, 9, 37, 19, 13, 21)_{\Theta(3,4,5)}$

under the action of the mapping $x \mapsto x + 3 \pmod{39}$, $\infty \mapsto \infty$.

K_{49} Let the vertex set be Z_{49} . The decompositions consist of

$(0, 27, 37, 26, 8, 6, 19, 47, 23, 30, 41)_{\Theta(1,2,9)},$
 $(14, 5, 6, 8, 3, 0, 15, 11, 27, 7, 24)_{\Theta(1,2,9)},$
 $(0, 26, 44, 41, 45, 28, 12, 10, 47, 22, 15)_{\Theta(1,3,8)},$
 $(24, 2, 3, 11, 4, 5, 19, 0, 31, 18, 8)_{\Theta(1,3,8)},$
 $(0, 26, 16, 46, 14, 11, 47, 32, 7, 42, 35)_{\Theta(1,4,7)},$
 $(23, 1, 2, 0, 4, 3, 8, 7, 15, 9, 19)_{\Theta(1,4,7)},$

$(0, 40, 43, 21, 24, 17, 19, 37, 13, 45, 6)_{\Theta(1,5,6)},$
 $(28, 0, 8, 3, 1, 12, 14, 6, 2, 15, 16)_{\Theta(1,5,6)},$
 $(0, 33, 13, 10, 34, 23, 2, 18, 22, 5, 27)_{\Theta(2,3,7)},$
 $(25, 0, 7, 6, 8, 10, 1, 4, 9, 23, 11)_{\Theta(2,3,7)},$
 $(0, 22, 36, 16, 10, 1, 3, 5, 20, 2, 39)_{\Theta(2,5,5)},$
 $(36, 0, 7, 8, 4, 1, 11, 9, 10, 2, 25)_{\Theta(2,5,5)},$
 $(0, 20, 28, 26, 39, 3, 35, 13, 29, 14, 21)_{\Theta(3,3,6)},$
 $(47, 0, 1, 5, 6, 18, 9, 28, 3, 29, 20)_{\Theta(3,3,6)},$
 $(0, 42, 13, 31, 16, 38, 35, 45, 17, 47, 7)_{\Theta(3,4,5)},$
 $(13, 0, 1, 2, 3, 8, 23, 19, 27, 7, 24)_{\Theta(3,4,5)}$

under the action of the mapping $x \mapsto x + 1 \pmod{49}$.

K_{57} Let the vertex set be Z_{57} . The decompositions consist of

$(0, 49, 36, 51, 16, 40, 53, 35, 46, 15, 17)_{\Theta(1,2,9)},$
 $(11, 15, 19, 35, 54, 52, 17, 10, 44, 47, 18)_{\Theta(1,2,9)},$
 $(47, 53, 10, 0, 40, 46, 16, 11, 49, 20, 9)_{\Theta(1,2,9)},$
 $(16, 15, 0, 31, 35, 30, 12, 26, 38, 45, 23)_{\Theta(1,2,9)},$
 $(13, 44, 53, 49, 52, 7, 18, 2, 3, 33, 21)_{\Theta(1,2,9)},$
 $(20, 35, 19, 41, 11, 13, 27, 51, 22, 32, 15)_{\Theta(1,2,9)},$
 $(0, 7, 25, 9, 4, 13, 33, 8, 39, 16, 54)_{\Theta(1,2,9)},$
 $(0, 16, 45, 24, 6, 35, 33, 15, 11, 37, 7)_{\Theta(1,3,8)},$
 $(17, 24, 55, 27, 31, 34, 15, 37, 50, 47, 22)_{\Theta(1,3,8)},$
 $(28, 23, 7, 1, 21, 53, 42, 0, 34, 49, 54)_{\Theta(1,3,8)},$
 $(12, 20, 52, 53, 50, 43, 32, 42, 15, 35, 41)_{\Theta(1,3,8)},$
 $(51, 50, 18, 23, 17, 2, 15, 40, 39, 49, 36)_{\Theta(1,3,8)},$
 $(20, 2, 10, 22, 42, 46, 3, 34, 17, 19, 11)_{\Theta(1,3,8)},$
 $(46, 7, 0, 41, 5, 1, 21, 30, 47, 2, 31)_{\Theta(1,3,8)},$
 $(0, 33, 54, 4, 3, 42, 26, 53, 15, 34, 41)_{\Theta(1,4,7)},$
 $(2, 43, 6, 16, 46, 5, 17, 11, 33, 44, 54)_{\Theta(1,4,7)},$
 $(13, 0, 48, 28, 51, 7, 19, 45, 38, 21, 4)_{\Theta(1,4,7)},$
 $(33, 45, 25, 16, 31, 47, 46, 8, 32, 55, 11)_{\Theta(1,4,7)},$
 $(12, 10, 40, 8, 50, 33, 49, 54, 36, 41, 43)_{\Theta(1,4,7)},$
 $(2, 11, 3, 23, 36, 22, 53, 32, 37, 55, 19)_{\Theta(1,4,7)},$
 $(34, 5, 20, 2, 33, 38, 16, 26, 37, 12, 3)_{\Theta(1,4,7)},$
 $(0, 11, 54, 21, 30, 4, 17, 48, 35, 50, 18)_{\Theta(1,5,6)},$
 $(24, 20, 45, 28, 42, 1, 16, 32, 2, 19, 17)_{\Theta(1,5,6)},$
 $(31, 8, 41, 29, 24, 52, 21, 1, 3, 23, 14)_{\Theta(1,5,6)},$
 $(51, 53, 29, 21, 6, 12, 13, 40, 18, 25, 28)_{\Theta(1,5,6)},$
 $(16, 4, 48, 14, 42, 41, 21, 22, 23, 47, 52)_{\Theta(1,5,6)},$
 $(55, 51, 2, 37, 3, 21, 41, 5, 15, 53, 7)_{\Theta(1,5,6)},$
 $(28, 4, 2, 20, 49, 10, 7, 56, 42, 30, 19)_{\Theta(1,5,6)},$
 $(0, 39, 52, 32, 8, 15, 34, 31, 26, 29, 21)_{\Theta(2,3,7)},$
 $(51, 8, 55, 44, 45, 42, 22, 20, 1, 5, 21)_{\Theta(2,3,7)},$
 $(12, 1, 42, 9, 38, 34, 10, 23, 5, 3, 49)_{\Theta(2,3,7)},$
 $(48, 47, 5, 29, 20, 25, 7, 0, 17, 1, 41)_{\Theta(2,3,7)},$
 $(49, 26, 38, 9, 3, 24, 36, 0, 31, 5, 34)_{\Theta(2,3,7)},$

$(48, 47, 26, 24, 22, 34, 6, 16, 50, 49, 4)_{\Theta(2,3,7)},$
 $(45, 9, 56, 37, 10, 41, 19, 13, 28, 7, 14)_{\Theta(2,3,7)},$
 $(0, 24, 30, 9, 53, 17, 14, 3, 1, 32, 13)_{\Theta(2,5,5)},$
 $(52, 4, 44, 39, 55, 16, 29, 9, 48, 49, 34)_{\Theta(2,5,5)},$
 $(37, 46, 9, 41, 25, 6, 44, 43, 40, 19, 24)_{\Theta(2,5,5)},$
 $(40, 36, 5, 26, 9, 50, 51, 35, 11, 41, 2)_{\Theta(2,5,5)},$
 $(38, 26, 24, 32, 52, 43, 51, 4, 0, 20, 19)_{\Theta(2,5,5)},$
 $(40, 21, 9, 7, 24, 49, 42, 28, 38, 47, 54)_{\Theta(2,5,5)},$
 $(0, 3, 8, 10, 33, 44, 56, 35, 46, 17, 32)_{\Theta(2,5,5)},$
 $(0, 41, 24, 25, 28, 9, 54, 38, 19, 2, 4)_{\Theta(3,3,6)},$
 $(7, 53, 33, 35, 14, 36, 38, 45, 40, 34, 44)_{\Theta(3,3,6)},$
 $(32, 36, 2, 54, 24, 49, 53, 19, 12, 55, 52)_{\Theta(3,3,6)},$
 $(24, 26, 53, 39, 47, 2, 3, 12, 6, 36, 1)_{\Theta(3,3,6)},$
 $(27, 55, 7, 25, 17, 6, 15, 0, 10, 23, 22)_{\Theta(3,3,6)},$
 $(36, 8, 34, 43, 40, 11, 19, 42, 23, 28, 14)_{\Theta(3,3,6)},$
 $(47, 0, 1, 46, 21, 20, 5, 13, 28, 49, 53)_{\Theta(3,3,6)},$
 $(0, 54, 29, 32, 30, 19, 20, 3, 25, 28, 49)_{\Theta(3,4,5)},$
 $(18, 45, 11, 6, 43, 42, 5, 54, 3, 10, 33)_{\Theta(3,4,5)},$
 $(39, 43, 6, 2, 49, 51, 41, 24, 55, 29, 10)_{\Theta(3,4,5)},$
 $(35, 22, 51, 8, 43, 28, 10, 25, 5, 14, 3)_{\Theta(3,4,5)},$
 $(47, 46, 35, 33, 41, 52, 9, 13, 21, 29, 2)_{\Theta(3,4,5)},$
 $(43, 40, 15, 47, 16, 12, 11, 52, 36, 19, 44)_{\Theta(3,4,5)},$
 $(43, 3, 8, 29, 26, 50, 12, 49, 44, 5, 47)_{\Theta(3,4,5)}$

under the action of the mapping $x \mapsto x + 3 \pmod{57}$.

K_{81} Let the vertex set be Z_{81} . The decompositions consist of

$(0, 23, 32, 38, 13, 9, 54, 70, 27, 67, 60)_{\Theta(1,2,9)},$
 $(28, 23, 56, 43, 77, 57, 63, 73, 53, 27, 1)_{\Theta(1,2,9)},$
 $(55, 18, 53, 14, 65, 28, 51, 24, 7, 31, 77)_{\Theta(1,2,9)},$
 $(40, 60, 31, 27, 22, 36, 17, 20, 44, 2, 8)_{\Theta(1,2,9)},$
 $(26, 36, 47, 24, 33, 74, 27, 52, 23, 40, 21)_{\Theta(1,2,9)},$
 $(19, 29, 37, 40, 13, 17, 30, 14, 28, 78, 74)_{\Theta(1,2,9)},$
 $(65, 12, 10, 77, 11, 78, 79, 46, 0, 18, 42)_{\Theta(1,2,9)},$
 $(40, 59, 41, 34, 50, 33, 72, 64, 13, 16, 5)_{\Theta(1,2,9)},$
 $(61, 68, 19, 12, 60, 13, 25, 3, 2, 27, 55)_{\Theta(1,2,9)},$
 $(30, 35, 27, 18, 11, 42, 21, 10, 46, 8, 58)_{\Theta(1,2,9)},$
 $(0, 11, 4, 77, 65, 43, 42, 50, 8, 41, 10)_{\Theta(1,3,8)},$
 $(4, 47, 36, 58, 27, 63, 7, 41, 21, 18, 37)_{\Theta(1,3,8)},$
 $(53, 59, 29, 52, 24, 7, 60, 45, 63, 54, 10)_{\Theta(1,3,8)},$
 $(4, 37, 59, 73, 1, 31, 50, 70, 64, 52, 57)_{\Theta(1,3,8)},$
 $(24, 70, 23, 28, 72, 47, 11, 78, 49, 65, 30)_{\Theta(1,3,8)},$
 $(71, 31, 74, 57, 43, 36, 15, 53, 41, 62, 58)_{\Theta(1,3,8)},$
 $(78, 7, 41, 28, 27, 59, 32, 50, 54, 14, 21)_{\Theta(1,3,8)},$
 $(58, 2, 29, 57, 56, 26, 36, 41, 54, 4, 19)_{\Theta(1,3,8)},$
 $(18, 65, 57, 19, 10, 28, 12, 35, 44, 63, 6)_{\Theta(1,3,8)},$
 $(17, 61, 15, 4, 48, 1, 3, 57, 45, 39, 52)_{\Theta(1,3,8)},$
 $(0, 65, 70, 36, 53, 26, 51, 42, 19, 46, 45)_{\Theta(1,4,7)},$

$(54, 16, 22, 15, 11, 49, 25, 9, 8, 61, 31)_{\Theta(1,4,7)},$
 $(25, 4, 69, 24, 52, 45, 78, 49, 10, 27, 30)_{\Theta(1,4,7)},$
 $(30, 70, 77, 40, 56, 48, 38, 14, 13, 9, 78)_{\Theta(1,4,7)},$
 $(5, 64, 59, 21, 23, 67, 54, 62, 79, 77, 38)_{\Theta(1,4,7)},$
 $(0, 29, 14, 27, 77, 31, 49, 58, 33, 26, 48)_{\Theta(1,4,7)},$
 $(8, 53, 58, 38, 35, 37, 25, 3, 27, 13, 48)_{\Theta(1,4,7)},$
 $(77, 73, 70, 67, 35, 31, 76, 65, 33, 68, 2)_{\Theta(1,4,7)},$
 $(55, 49, 47, 41, 62, 32, 60, 54, 15, 36, 51)_{\Theta(1,4,7)},$
 $(0, 51, 41, 32, 62, 19, 9, 63, 5, 61, 14)_{\Theta(1,4,7)},$
 $(0, 24, 62, 52, 38, 20, 76, 71, 60, 14, 2)_{\Theta(1,5,6)},$
 $(30, 18, 9, 16, 19, 33, 67, 1, 54, 47, 63)_{\Theta(1,5,6)},$
 $(12, 43, 76, 72, 62, 5, 45, 74, 71, 50, 10)_{\Theta(1,5,6)},$
 $(51, 73, 64, 9, 23, 4, 28, 49, 20, 53, 8)_{\Theta(1,5,6)},$
 $(7, 32, 29, 52, 12, 1, 13, 4, 66, 8, 25)_{\Theta(1,5,6)},$
 $(19, 55, 51, 48, 49, 29, 76, 33, 50, 12, 56)_{\Theta(1,5,6)},$
 $(71, 58, 43, 63, 65, 16, 29, 37, 55, 57, 56)_{\Theta(1,5,6)},$
 $(1, 30, 72, 42, 29, 2, 36, 44, 40, 77, 21)_{\Theta(1,5,6)},$
 $(76, 41, 49, 15, 62, 21, 46, 30, 3, 66, 72)_{\Theta(1,5,6)},$
 $(29, 24, 23, 70, 59, 50, 14, 65, 33, 58, 66)_{\Theta(1,5,6)},$
 $(0, 40, 68, 78, 37, 24, 10, 6, 50, 17, 4)_{\Theta(2,3,7)},$
 $(70, 60, 69, 5, 24, 50, 64, 51, 33, 16, 59)_{\Theta(2,3,7)},$
 $(73, 1, 21, 75, 16, 49, 44, 25, 30, 19, 74)_{\Theta(2,3,7)},$
 $(33, 5, 41, 60, 77, 7, 44, 51, 56, 59, 57)_{\Theta(2,3,7)},$
 $(63, 14, 20, 31, 29, 16, 34, 76, 5, 66, 41)_{\Theta(2,3,7)},$
 $(64, 79, 5, 53, 4, 55, 48, 59, 58, 10, 44)_{\Theta(2,3,7)},$
 $(23, 47, 64, 27, 17, 57, 71, 30, 42, 77, 24)_{\Theta(2,3,7)},$
 $(17, 27, 46, 67, 57, 13, 34, 4, 69, 47, 70)_{\Theta(2,3,7)},$
 $(1, 22, 45, 70, 78, 36, 15, 43, 16, 41, 72)_{\Theta(2,3,7)},$
 $(51, 41, 57, 36, 62, 3, 42, 74, 5, 23, 65)_{\Theta(2,3,7)},$
 $(0, 72, 9, 29, 59, 6, 31, 50, 7, 26, 5)_{\Theta(2,5,5)},$
 $(55, 78, 70, 72, 67, 14, 71, 79, 46, 58, 2)_{\Theta(2,5,5)},$
 $(79, 11, 0, 8, 72, 68, 27, 69, 40, 10, 7)_{\Theta(2,5,5)},$
 $(53, 63, 17, 9, 16, 32, 51, 62, 75, 54, 31)_{\Theta(2,5,5)},$
 $(7, 54, 3, 62, 56, 33, 76, 16, 10, 73, 18)_{\Theta(2,5,5)},$
 $(37, 9, 48, 35, 20, 38, 11, 8, 22, 43, 24)_{\Theta(2,5,5)},$
 $(0, 66, 56, 46, 5, 53, 16, 61, 29, 30, 33)_{\Theta(2,5,5)},$
 $(26, 0, 13, 65, 6, 30, 38, 37, 76, 77, 1)_{\Theta(2,5,5)},$
 $(64, 20, 17, 71, 39, 76, 40, 78, 72, 25, 8)_{\Theta(2,5,5)},$
 $(27, 16, 74, 47, 1, 28, 0, 54, 80, 7, 38)_{\Theta(2,5,5)},$
 $(0, 63, 14, 55, 54, 73, 22, 15, 58, 71, 33)_{\Theta(3,3,6)},$
 $(74, 56, 53, 9, 22, 59, 18, 31, 43, 46, 71)_{\Theta(3,3,6)},$
 $(32, 66, 43, 72, 31, 64, 45, 49, 35, 13, 78)_{\Theta(3,3,6)},$
 $(71, 45, 73, 47, 32, 42, 17, 62, 53, 43, 74)_{\Theta(3,3,6)},$
 $(51, 73, 1, 22, 60, 49, 44, 12, 45, 53, 58)_{\Theta(3,3,6)},$
 $(16, 5, 77, 36, 44, 52, 23, 18, 60, 24, 1)_{\Theta(3,3,6)},$
 $(15, 42, 49, 26, 68, 25, 41, 65, 19, 63, 59)_{\Theta(3,3,6)},$
 $(45, 63, 4, 58, 73, 28, 31, 25, 44, 62, 74)_{\Theta(3,3,6)},$

$(59, 46, 39, 7, 53, 72, 29, 75, 55, 30, 45)_{\Theta(3,3,6)},$
 $(18, 43, 77, 61, 17, 34, 0, 57, 36, 59, 11)_{\Theta(3,3,6)},$
 $(0, 19, 28, 14, 26, 63, 34, 8, 25, 38, 50)_{\Theta(3,4,5)},$
 $(27, 6, 11, 39, 58, 56, 38, 24, 7, 41, 49)_{\Theta(3,4,5)},$
 $(75, 10, 71, 51, 55, 65, 28, 5, 58, 52, 61)_{\Theta(3,4,5)},$
 $(73, 34, 70, 48, 75, 36, 61, 52, 76, 17, 23)_{\Theta(3,4,5)},$
 $(78, 24, 60, 36, 69, 2, 40, 26, 46, 13, 39)_{\Theta(3,4,5)},$
 $(47, 11, 77, 56, 38, 73, 24, 23, 0, 76, 34)_{\Theta(3,4,5)},$
 $(48, 77, 12, 62, 29, 75, 74, 78, 44, 71, 38)_{\Theta(3,4,5)},$
 $(17, 6, 16, 52, 49, 65, 27, 58, 48, 61, 60)_{\Theta(3,4,5)},$
 $(46, 75, 42, 35, 71, 52, 64, 20, 16, 39, 31)_{\Theta(3,4,5)},$
 $(55, 72, 21, 77, 36, 14, 78, 48, 38, 67, 74)_{\Theta(3,4,5)}$

under the action of the mapping $x \mapsto x + 3 \pmod{81}$. □

Proof of Lemma 3.6

$K_{8,12}$ Let the vertex set be $\{0, 1, \dots, 19\}$ partitioned into $\{0, 1, \dots, 7\}$ and $\{8, 9, \dots, 19\}$. The decompositions consist of

$(0, 2, 8, 9, 10, 1, 14, 3, 13, 4, 12)_{\Theta(2,2,8)},$
 $(0, 2, 8, 9, 1, 10, 11, 6, 16, 5, 18)_{\Theta(2,4,6)},$
 $(0, 2, 8, 1, 9, 10, 3, 13, 11, 7, 18)_{\Theta(4,4,4)}$

under the action of the mapping $x \mapsto x + 1 \pmod{8}$ for $x < 8$, $x \mapsto (x - 8 + 3 \pmod{12}) + 8$ for $x \geq 8$.

$K_{8,8,8}$ Let the vertex set be Z_{24} partitioned according to residue classes modulo 3. The decompositions consist of

$(0, 1, 2, 4, 6, 5, 7, 11, 3, 8, 12)_{\Theta(1,2,9)},$
 $(0, 7, 14, 10, 2, 13, 8, 22, 6, 23, 12)_{\Theta(1,2,9)},$
 $(0, 1, 2, 3, 4, 5, 7, 11, 6, 13, 8)_{\Theta(1,3,8)},$
 $(0, 13, 8, 18, 10, 2, 6, 17, 4, 14, 21)_{\Theta(1,3,8)},$
 $(0, 1, 2, 3, 8, 4, 5, 7, 9, 16, 6)_{\Theta(1,4,7)},$
 $(0, 13, 8, 4, 23, 11, 19, 3, 20, 6, 2)_{\Theta(1,4,7)},$
 $(0, 1, 2, 3, 7, 5, 8, 12, 4, 6, 11)_{\Theta(1,5,6)},$
 $(0, 7, 10, 2, 1, 12, 11, 4, 23, 6, 20)_{\Theta(1,5,6)},$
 $(0, 1, 2, 4, 3, 5, 6, 13, 8, 10, 14)_{\Theta(2,3,7)},$
 $(0, 1, 8, 11, 15, 13, 5, 19, 3, 20, 6)_{\Theta(2,3,7)},$
 $(0, 1, 2, 4, 3, 8, 9, 7, 5, 10, 12)_{\Theta(2,5,5)},$
 $(0, 1, 8, 10, 2, 9, 5, 14, 3, 22, 11)_{\Theta(2,5,5)},$
 $(0, 1, 2, 3, 4, 5, 7, 6, 11, 13, 8)_{\Theta(3,3,6)},$
 $(0, 1, 8, 12, 16, 6, 11, 21, 14, 4, 17)_{\Theta(3,3,6)},$
 $(0, 1, 2, 3, 4, 5, 9, 7, 6, 16, 8)_{\Theta(3,4,5)},$
 $(0, 1, 5, 12, 8, 4, 14, 19, 17, 6, 20)_{\Theta(3,4,5)}$

under the action of the mapping $x \mapsto x + 3 \pmod{24}$.

$K_{8,8,8,8}$ Let the vertex set be Z_{32} partitioned according to residue classes modulo 4. The decompositions consist of

$(0, 2, 3, 5, 11, 1, 8, 21, 4, 22, 13)_{\Theta(1,2,9)},$
 $(0, 2, 1, 7, 3, 10, 19, 4, 14, 27, 13)_{\Theta(1,3,8)},$
 $(0, 2, 1, 4, 9, 6, 15, 5, 16, 3, 17)_{\Theta(1,4,7)},$
 $(0, 2, 1, 4, 9, 15, 7, 16, 26, 5, 19)_{\Theta(1,5,6)},$
 $(0, 2, 3, 5, 11, 7, 9, 22, 8, 23, 12)_{\Theta(2,3,7)},$
 $(0, 2, 3, 5, 7, 1, 11, 13, 20, 6, 17)_{\Theta(2,5,5)},$
 $(0, 2, 1, 4, 5, 11, 7, 20, 6, 23, 12)_{\Theta(3,3,6)},$
 $(0, 2, 1, 4, 5, 11, 20, 7, 17, 6, 19)_{\Theta(3,4,5)}.$

under the action of the mapping $x \mapsto x + 1 \pmod{32}$.

$K_{8,8,8,24}$ Let the vertex set be Z_{48} partitioned according to residue classes 0, 1, 2 and $\{3, 4, 5\}$ modulo 6. The decompositions consist of

$(0, 35, 32, 31, 29, 7, 2, 10, 6, 4, 36)_{\Theta(1,2,9)},$
 $(27, 20, 43, 42, 19, 9, 38, 39, 2, 1, 6)_{\Theta(1,2,9)},$
 $(24, 27, 2, 5, 25, 10, 7, 15, 18, 8, 1)_{\Theta(1,2,9)},$
 $(11, 20, 18, 2, 35, 12, 17, 44, 40, 8, 46)_{\Theta(1,2,9)},$
 $(46, 12, 1, 7, 3, 36, 28, 18, 19, 5, 8)_{\Theta(1,2,9)},$
 $(31, 40, 18, 45, 14, 43, 24, 32, 46, 44, 13)_{\Theta(1,2,9)},$
 $(0, 17, 7, 27, 36, 45, 25, 46, 18, 29, 13)_{\Theta(1,2,9)},$
 $(19, 11, 32, 34, 14, 25, 27, 6, 26, 39, 44)_{\Theta(1,2,9)},$
 $(0, 22, 10, 6, 33, 19, 32, 41, 42, 38, 31)_{\Theta(1,3,8)},$
 $(15, 38, 26, 0, 44, 12, 25, 24, 3, 30, 19)_{\Theta(1,3,8)},$
 $(22, 43, 36, 17, 25, 15, 37, 41, 7, 38, 28)_{\Theta(1,3,8)},$
 $(23, 12, 30, 35, 6, 31, 11, 8, 22, 44, 3)_{\Theta(1,3,8)},$
 $(20, 12, 5, 14, 15, 13, 23, 7, 27, 26, 9)_{\Theta(1,3,8)},$
 $(17, 19, 20, 40, 44, 30, 39, 24, 43, 35, 8)_{\Theta(1,3,8)},$
 $(36, 43, 8, 4, 16, 14, 46, 6, 1, 26, 39)_{\Theta(1,3,8)},$
 $(21, 13, 18, 16, 37, 4, 44, 1, 2, 17, 30)_{\Theta(1,3,8)},$
 $(0, 22, 29, 20, 43, 13, 46, 7, 23, 44, 30)_{\Theta(1,4,7)},$
 $(32, 5, 4, 31, 36, 39, 38, 6, 22, 12, 43)_{\Theta(1,4,7)},$
 $(15, 24, 44, 29, 31, 7, 4, 37, 39, 18, 23)_{\Theta(1,4,7)},$
 $(14, 29, 3, 31, 38, 10, 8, 36, 23, 0, 37)_{\Theta(1,4,7)},$
 $(6, 10, 14, 9, 12, 39, 1, 35, 42, 28, 2)_{\Theta(1,4,7)},$
 $(32, 42, 16, 26, 43, 29, 25, 44, 1, 33, 20)_{\Theta(1,4,7)},$
 $(26, 30, 29, 7, 10, 3, 20, 18, 45, 19, 33)_{\Theta(1,4,7)},$
 $(27, 18, 12, 23, 43, 31, 20, 19, 32, 46, 37)_{\Theta(1,4,7)},$
 $(0, 14, 16, 1, 44, 12, 40, 37, 36, 5, 25)_{\Theta(1,5,6)},$
 $(29, 31, 20, 3, 13, 42, 18, 40, 36, 23, 2)_{\Theta(1,5,6)},$
 $(35, 2, 31, 23, 18, 39, 42, 41, 7, 29, 19)_{\Theta(1,5,6)},$
 $(22, 14, 32, 11, 30, 39, 24, 27, 25, 46, 36)_{\Theta(1,5,6)},$
 $(27, 42, 14, 15, 24, 21, 20, 35, 38, 0, 7)_{\Theta(1,5,6)},$
 $(34, 32, 43, 15, 19, 35, 38, 13, 45, 30, 10)_{\Theta(1,5,6)},$
 $(27, 8, 19, 36, 13, 22, 32, 23, 0, 20, 40)_{\Theta(1,5,6)},$
 $(20, 13, 19, 4, 25, 39, 12, 46, 1, 14, 18)_{\Theta(1,5,6)},$
 $(0, 28, 43, 1, 32, 37, 17, 36, 2, 18, 14)_{\Theta(2,3,7)},$

$(35, 12, 2, 37, 23, 30, 32, 16, 38, 18, 22)_{\Theta(2,3,7)},$
 $(5, 8, 31, 12, 29, 43, 22, 0, 35, 44, 33)_{\Theta(2,3,7)},$
 $(10, 7, 24, 13, 22, 18, 33, 42, 45, 12, 9)_{\Theta(2,3,7)},$
 $(42, 40, 19, 3, 32, 15, 2, 24, 22, 31, 12)_{\Theta(2,3,7)},$
 $(28, 41, 26, 8, 1, 12, 11, 2, 3, 20, 25)_{\Theta(2,3,7)},$
 $(6, 23, 19, 29, 26, 33, 37, 38, 28, 25, 44)_{\Theta(2,3,7)},$
 $(7, 2, 45, 33, 13, 15, 31, 44, 36, 43, 9)_{\Theta(2,3,7)},$
 $(0, 8, 21, 16, 24, 19, 4, 41, 25, 9, 7)_{\Theta(2,5,5)},$
 $(30, 42, 29, 2, 13, 8, 33, 14, 21, 32, 3)_{\Theta(2,5,5)},$
 $(2, 5, 42, 6, 21, 7, 36, 29, 43, 12, 38)_{\Theta(2,5,5)},$
 $(44, 15, 30, 37, 34, 36, 43, 5, 20, 17, 7)_{\Theta(2,5,5)},$
 $(12, 36, 37, 1, 23, 32, 15, 34, 44, 42, 22)_{\Theta(2,5,5)},$
 $(4, 32, 1, 42, 17, 25, 34, 31, 9, 12, 16)_{\Theta(2,5,5)},$
 $(30, 22, 43, 35, 7, 11, 8, 20, 28, 2, 31)_{\Theta(2,5,5)},$
 $(45, 5, 7, 1, 26, 27, 24, 2, 37, 4, 32)_{\Theta(2,5,5)},$
 $(0, 45, 26, 43, 1, 24, 34, 18, 3, 20, 30)_{\Theta(3,3,6)},$
 $(8, 45, 16, 7, 43, 44, 24, 19, 30, 4, 26)_{\Theta(3,3,6)},$
 $(0, 22, 45, 18, 41, 1, 19, 34, 14, 46, 8)_{\Theta(3,3,6)},$
 $(10, 20, 7, 23, 31, 9, 12, 40, 38, 11, 13)_{\Theta(3,3,6)},$
 $(34, 31, 42, 8, 38, 3, 24, 5, 19, 23, 0)_{\Theta(3,3,6)},$
 $(10, 38, 19, 33, 13, 45, 25, 35, 2, 31, 18)_{\Theta(3,3,6)},$
 $(2, 18, 17, 26, 7, 29, 27, 36, 39, 31, 20)_{\Theta(3,3,6)},$
 $(24, 1, 20, 29, 11, 42, 23, 44, 41, 36, 45)_{\Theta(3,3,6)},$
 $(0, 44, 45, 6, 14, 25, 3, 15, 20, 21, 13)_{\Theta(3,4,5)},$
 $(21, 7, 25, 24, 1, 34, 14, 36, 4, 8, 23)_{\Theta(3,4,5)},$
 $(39, 36, 8, 31, 1, 22, 20, 18, 27, 43, 46)_{\Theta(3,4,5)},$
 $(25, 22, 18, 7, 27, 38, 30, 12, 23, 44, 0)_{\Theta(3,4,5)},$
 $(29, 18, 1, 46, 24, 22, 43, 12, 15, 36, 11)_{\Theta(3,4,5)},$
 $(12, 20, 38, 23, 31, 35, 25, 46, 8, 6, 41)_{\Theta(3,4,5)},$
 $(4, 23, 44, 24, 26, 10, 1, 0, 29, 31, 32)_{\Theta(3,4,5)},$
 $(14, 32, 33, 19, 23, 37, 29, 39, 26, 7, 46)_{\Theta(3,4,5)}$

under the action of the mapping $x \mapsto x + 6 \pmod{48}$. □

D Theta graphs of 13 edges

Proof of Lemma 3.7

K_{13} Let the vertex set be $Z_{12} \cup \{\infty\}$. The decompositions consist of the graphs

$(0, 1, 2, 3, 4, 8, 5, 7, 6, 10, \infty, 11)_{\Theta(1,2,10)},$
 $(0, 5, 10, 6, 3, 9, 1, \infty, 4, 11, 7, 2)_{\Theta(1,2,10)},$
 $(1, 0, 2, 3, 4, 6, 8, 7, 10, 5, 11, \infty)_{\Theta(1,3,9)},$
 $(0, 5, 6, 2, 4, 11, 1, 9, \infty, 10, 3, 7)_{\Theta(1,3,9)},$
 $(0, 1, 2, 3, 4, 6, 5, 7, 10, \infty, 8, 11)_{\Theta(1,4,8)},$
 $(0, 5, 4, 2, 9, 7, 1, 10, 6, 11, 3, \infty)_{\Theta(1,4,8)},$
 $(1, 0, 2, 3, 4, 6, 5, 7, 10, \infty, 11, 8)_{\Theta(1,5,7)},$
 $(0, 5, 7, 1, 4, \infty, 10, 2, 9, 6, 11, 3)_{\Theta(1,5,7)},$

$(0, 1, 2, 3, 4, 7, 5, 6, 9, 10, \infty, 11)_{\Theta(1,6,6)},$
 $(0, 5, 4, 1, 6, 8, \infty, 7, 3, 10, 2, 11)_{\Theta(1,6,6)},$
 $(0, 1, 2, 3, 4, 5, 8, 6, 7, 10, \infty, 11)_{\Theta(2,2,9)},$
 $(0, 1, 5, 6, 11, 4, \infty, 9, 3, 7, 2, 10)_{\Theta(2,2,9)},$
 $(0, 1, 2, 3, 4, 5, 7, 9, 6, 10, 11, \infty)_{\Theta(2,3,8)},$
 $(0, 1, 6, 4, 5, 10, 3, 8, \infty, 2, 11, 7)_{\Theta(2,3,8)},$
 $(0, 1, 2, 3, 4, 5, 6, 7, 9, 11, \infty, 10)_{\Theta(2,4,7)},$
 $(0, 1, 7, 5, 8, \infty, 4, 2, 11, 3, 10, 6)_{\Theta(2,4,7)},$
 $(0, 1, 2, 3, 4, 5, 7, 6, 8, \infty, 10, 11)_{\Theta(2,5,6)},$
 $(0, 1, 4, 5, 9, 2, 10, 7, 3, 6, 11, \infty)_{\Theta(2,5,6)},$
 $(1, 0, 2, 3, 4, 5, 6, 8, 10, 7, 11, \infty)_{\Theta(3,3,7)},$
 $(0, 1, 4, 3, 6, 9, 7, 2, 10, \infty, 5, 11)_{\Theta(3,3,7)},$
 $(0, 1, 2, 3, 4, 5, 6, 7, 8, 11, \infty, 10)_{\Theta(3,4,6)},$
 $(0, 5, 10, 3, 9, 4, \infty, 6, 2, 11, 7, 1)_{\Theta(3,4,6)},$
 $(1, 0, 2, 3, 4, 5, 7, 8, 6, 11, \infty, 10)_{\Theta(3,5,5)},$
 $(0, 1, 2, 5, 6, 10, 7, 11, \infty, 9, 3, 8)_{\Theta(3,5,5)},$
 $(0, 1, 2, 3, 4, 5, 6, 9, 7, 10, \infty, 11)_{\Theta(4,4,5)},$
 $(0, 1, 3, 11, 6, 4, 5, 7, 10, 2, 8, \infty)_{\Theta(4,4,5)}$

under the action of the mapping $x \mapsto x + 4 \pmod{12}$, $\infty \mapsto \infty$.

K_{14} Let the vertex set be Z_{14} . The decompositions consist of

$(0, 1, 2, 3, 5, 8, 4, 9, 13, 6, 12, 7)_{\Theta(1,2,10)},$
 $(0, 1, 2, 5, 4, 3, 8, 13, 6, 12, 9, 7)_{\Theta(1,3,9)},$
 $(0, 1, 2, 5, 6, 4, 10, 3, 12, 9, 11, 7)_{\Theta(1,4,8)},$
 $(0, 1, 2, 5, 3, 4, 8, 12, 7, 13, 6, 11)_{\Theta(1,5,7)},$
 $(0, 1, 2, 5, 3, 4, 8, 6, 11, 7, 12, 9)_{\Theta(1,6,6)},$
 $(0, 1, 2, 3, 4, 9, 12, 6, 13, 5, 10, 11)_{\Theta(2,2,9)},$
 $(0, 1, 2, 3, 5, 6, 7, 13, 4, 9, 12, 8)_{\Theta(2,3,8)},$
 $(0, 1, 2, 3, 5, 4, 8, 12, 7, 13, 6, 11)_{\Theta(2,4,7)},$
 $(0, 1, 2, 3, 5, 4, 8, 6, 11, 7, 12, 9)_{\Theta(2,5,6)},$
 $(0, 1, 2, 3, 4, 7, 8, 13, 9, 12, 5, 6)_{\Theta(3,3,7)},$
 $(0, 1, 2, 3, 4, 7, 8, 5, 13, 9, 12, 6)_{\Theta(3,4,6)},$
 $(0, 1, 2, 3, 4, 7, 12, 9, 8, 13, 6, 5)_{\Theta(3,5,5)},$
 $(0, 1, 2, 3, 4, 5, 12, 6, 10, 13, 9, 7)_{\Theta(4,4,5)}$

under the action of the mapping $x \mapsto x + 2 \pmod{14}$.

K_{26} Let the vertex set be $Z_{25} \cup \{\infty\}$. The decompositions consist of

$(\infty, 5, 2, 6, 7, 23, 17, 21, 3, 14, 20, 1)_{\Theta(1,2,10)},$
 $(6, 18, 16, 15, 19, 10, 12, 23, 4, 7, 21, 1)_{\Theta(1,2,10)},$
 $(22, 15, 16, 9, 4, 12, 7, 14, 21, 19, 17, 5)_{\Theta(1,2,10)},$
 $(18, 14, 15, 17, 0, 5, 19, 16, 4, 13, 3, 6)_{\Theta(1,2,10)},$
 $(20, 3, 8, 13, 14, 4, \infty, 18, 22, 7, 16, 5)_{\Theta(1,2,10)},$

$(\infty, 14, 11, 1, 6, 23, 22, 2, 15, 21, 5, 8)_{\Theta(1,4,8)},$
 $(22, 12, 16, 9, 19, 3, 15, 7, 5, 24, 23, 21)_{\Theta(1,4,8)},$
 $(10, 8, 21, 7, 11, 9, 4, 20, 16, 3, 14, 22)_{\Theta(1,4,8)},$
 $(11, 14, 24, 3, 10, 12, 9, 22, 18, 2, 5, 0)_{\Theta(1,4,8)},$
 $(17, 19, 10, 18, 3, \infty, 15, 0, 1, 8, 13, 21)_{\Theta(1,4,8)},$
 $(\infty, 1, 3, 11, 5, 15, 16, 22, 7, 6, 20, 12)_{\Theta(1,6,6)},$
 $(19, 13, 6, 3, 20, 23, 9, 21, 14, 22, 4, 15)_{\Theta(1,6,6)},$
 $(18, 13, 11, 14, 6, 15, 12, 2, 21, 0, 20, 4)_{\Theta(1,6,6)},$
 $(18, 22, 0, 12, 21, 17, 15, 8, 9, 24, 20, 19)_{\Theta(1,6,6)},$
 $(8, 20, 2, 7, 19, 24, \infty, 6, 11, 23, 12, 14)_{\Theta(1,6,6)},$
 $(\infty, 23, 9, 15, 8, 4, 22, 19, 14, 10, 0, 1)_{\Theta(2,2,9)},$
 $(17, 6, 1, 0, 15, 18, 2, 13, 11, 9, 10, 7)_{\Theta(2,2,9)},$
 $(19, 3, 9, 5, 17, 18, 1, 13, 7, 22, 14, 23)_{\Theta(2,2,9)},$
 $(5, 8, 12, 1, 14, 21, 10, 22, 2, 16, 6, 15)_{\Theta(2,2,9)},$
 $(21, 6, 2, 9, 4, 3, 13, 0, 5, 24, 12, \infty)_{\Theta(2,2,9)},$
 $(\infty, 21, 19, 0, 14, 11, 12, 24, 15, 17, 5, 6)_{\Theta(2,3,8)},$
 $(22, 8, 17, 14, 4, 12, 6, 1, 10, 21, 0, 19)_{\Theta(2,3,8)},$
 $(9, 14, 8, 12, 19, 5, 23, 13, 17, 3, 0, 15)_{\Theta(2,3,8)},$
 $(6, 20, 2, 22, 3, 17, 18, 5, 10, 8, 13, 1)_{\Theta(2,3,8)},$
 $(9, 1, 18, 6, 3, 21, 4, 2, 5, 22, \infty, 8)_{\Theta(2,3,8)},$
 $(\infty, 18, 11, 20, 8, 9, 22, 2, 21, 19, 15, 12)_{\Theta(2,4,7)},$
 $(3, 22, 20, 8, 4, 12, 14, 17, 19, 5, 11, 21)_{\Theta(2,4,7)},$
 $(0, 23, 11, 20, 12, 13, 1, 10, 19, 9, 16, 21)_{\Theta(2,4,7)},$
 $(14, 3, 9, 6, 10, 0, 2, 18, 20, 19, 16, 7)_{\Theta(2,4,7)},$
 $(8, 1, 22, \infty, 19, 12, 16, 4, 10, 17, 5, 23)_{\Theta(2,4,7)},$
 $(\infty, 21, 23, 11, 6, 2, 18, 19, 14, 1, 16, 15)_{\Theta(2,5,6)},$
 $(15, 4, 11, 5, 13, 9, 10, 6, 20, 22, 2, 14)_{\Theta(2,5,6)},$
 $(12, 13, 20, 15, 3, 21, 7, 9, 8, 23, 14, 10)_{\Theta(2,5,6)},$
 $(13, 12, 21, 17, 3, 16, 19, 8, 14, 5, 10, 22)_{\Theta(2,5,6)},$
 $(17, 8, 10, 16, 22, 24, 7, \infty, 15, 4, 21, 19)_{\Theta(2,5,6)},$
 $(\infty, 7, 5, 16, 11, 13, 6, 23, 20, 8, 12, 14)_{\Theta(3,4,6)},$
 $(15, 0, 11, 8, 2, 22, 7, 21, 17, 6, 19, 1)_{\Theta(3,4,6)},$
 $(14, 1, 16, 6, 2, 19, 13, 4, 7, 15, 0, 18)_{\Theta(3,4,6)},$
 $(3, 24, 17, 8, 4, 18, 20, 13, 7, 5, 10, 19)_{\Theta(3,4,6)},$
 $(0, 4, 22, \infty, 19, 5, 21, 24, 3, 8, 7, 1)_{\Theta(3,4,6)},$
 $(\infty, 17, 8, 13, 5, 10, 20, 19, 9, 4, 18, 6)_{\Theta(4,4,5)},$
 $(21, 19, 16, 14, 7, 17, 15, 13, 12, 5, 9, 6)_{\Theta(4,4,5)},$
 $(4, 5, 10, 13, 21, 8, 2, 1, 12, 15, 16, 18)_{\Theta(4,4,5)},$
 $(14, 11, 17, 12, 8, 21, 6, 0, 5, 19, 9, 18)_{\Theta(4,4,5)},$
 $(16, 3, 22, 5, 10, 24, 23, 12, \infty, 2, 17, 18)_{\Theta(4,4,5)}$

under the action of the mapping $x \mapsto x + 5 \pmod{25}$, $\infty \mapsto \infty$. □

Proof of Lemma 3.8

$K_{13,13}$ Let the vertex set be Z_{26} partitioned according to residue classes modulo 2. The decompositions consist of

$$\begin{aligned}
&(0, 17, 11, 4, 1, 2, 5, 8, 13, 18, 7, 24)_{\Theta(1,3,9)}, \\
&(0, 11, 21, 8, 17, 10, 3, 4, 9, 16, 5, 14)_{\Theta(1,5,7)}, \\
&(0, 19, 11, 14, 21, 2, 1, 12, 15, 16, 23, 10)_{\Theta(3,3,7)}, \\
&(0, 1, 3, 8, 15, 2, 7, 10, 11, 12, 19, 18)_{\Theta(3,5,5)}
\end{aligned}$$

under the action of the mapping $x \mapsto x + 2 \pmod{26}$.

$K_{13,13,13}$ Let the vertex set be Z_{39} partitioned according to residue classes modulo 3. The decompositions consist of

$$\begin{aligned}
&(0, 1, 5, 2, 9, 17, 3, 13, 26, 6, 28, 12)_{\Theta(1,2,10)}, \\
&(0, 1, 2, 6, 11, 7, 15, 26, 3, 17, 4, 21)_{\Theta(1,4,8)}, \\
&(0, 1, 2, 6, 11, 3, 14, 7, 23, 9, 28, 18)_{\Theta(1,6,6)}, \\
&(0, 1, 2, 5, 7, 15, 4, 14, 28, 8, 30, 17)_{\Theta(2,2,9)}, \\
&(0, 1, 2, 4, 9, 7, 17, 3, 14, 27, 5, 21)_{\Theta(2,3,8)}, \\
&(0, 1, 2, 4, 9, 17, 7, 27, 10, 35, 22, 11)_{\Theta(2,4,7)}, \\
&(0, 1, 2, 4, 9, 16, 24, 10, 23, 6, 26, 12)_{\Theta(2,5,6)}, \\
&(0, 1, 2, 6, 7, 8, 18, 11, 3, 17, 4, 20)_{\Theta(3,4,6)}, \\
&(0, 1, 2, 3, 8, 4, 12, 23, 10, 24, 37, 17)_{\Theta(4,4,5)}
\end{aligned}$$

under the action of the mapping $x \mapsto x + 1 \pmod{39}$.

$K_{13,13,13,13}$ Let the vertex set be $\{0, 1, \dots, 51\}$ partitioned into $\{3j + i : j = 0, 1, \dots, 12\}$, $i = 0, 1, 2$, and $\{39, 40, \dots, 51\}$. The decompositions consist of

$$\begin{aligned}
&(0, 11, 49, 34, 41, 2, 43, 27, 44, 20, 16, 3)_{\Theta(1,2,10)}, \\
&(49, 1, 2, 4, 6, 13, 3, 17, 0, 16, 36, 41)_{\Theta(1,2,10)}, \\
&(0, 38, 46, 12, 1, 13, 23, 9, 14, 6, 29, 47)_{\Theta(1,4,8)}, \\
&(7, 43, 14, 44, 16, 49, 11, 15, 32, 12, 51, 6)_{\Theta(1,4,8)}, \\
&(0, 49, 14, 6, 28, 23, 36, 41, 37, 8, 46, 38)_{\Theta(1,6,6)}, \\
&(1, 17, 33, 37, 35, 41, 36, 2, 13, 42, 9, 44)_{\Theta(1,6,6)}, \\
&(0, 12, 5, 22, 11, 30, 14, 27, 42, 38, 45, 13)_{\Theta(2,2,9)}, \\
&(21, 42, 13, 7, 39, 15, 19, 44, 8, 10, 49, 2)_{\Theta(2,2,9)}, \\
&(0, 46, 20, 41, 1, 5, 37, 6, 10, 24, 26, 13)_{\Theta(2,3,8)}, \\
&(34, 28, 6, 45, 27, 11, 1, 39, 4, 40, 10, 44)_{\Theta(2,3,8)}, \\
&(0, 44, 29, 39, 7, 2, 25, 14, 6, 26, 19, 20)_{\Theta(2,4,7)}, \\
&(21, 11, 34, 19, 42, 15, 39, 1, 18, 40, 7, 41)_{\Theta(2,4,7)}, \\
&(0, 25, 47, 23, 12, 40, 15, 17, 19, 18, 41, 38)_{\Theta(2,5,6)}, \\
&(10, 45, 14, 18, 25, 20, 0, 24, 41, 1, 47, 8)_{\Theta(2,5,6)}, \\
&(0, 31, 26, 48, 46, 17, 41, 2, 30, 38, 40, 14)_{\Theta(3,4,6)}, \\
&(38, 44, 6, 10, 19, 35, 30, 9, 8, 33, 45, 0)_{\Theta(3,4,6)}, \\
&(0, 45, 8, 48, 10, 5, 47, 21, 28, 35, 40, 38)_{\Theta(4,4,5)}, \\
&(9, 7, 19, 32, 30, 28, 24, 41, 8, 25, 11, 43)_{\Theta(4,4,5)}
\end{aligned}$$

under the action of the mapping $x \mapsto x + 1 \pmod{39}$ for $x < 39$, $x \mapsto (x + 1 \pmod{13}) + 39$ for $x \geq 39$.

$K_{13,13,13,13,13}$ Let the vertex set be Z_{65} partitioned according to residue class modulo 5. The decompositions consist of

$(0, 17, 29, 22, 48, 11, 34, 23, 61, 43, 19, 25)_{\Theta(1,2,10)},$
 $(27, 29, 60, 40, 47, 3, 19, 0, 4, 7, 6, 15)_{\Theta(1,2,10)},$
 $(0, 9, 2, 20, 13, 32, 44, 61, 19, 11, 63, 12)_{\Theta(1,4,8)},$
 $(17, 1, 6, 0, 22, 16, 40, 3, 37, 8, 46, 20)_{\Theta(1,4,8)},$
 $(0, 43, 7, 53, 2, 19, 10, 39, 16, 44, 55, 37)_{\Theta(1,6,6)},$
 $(17, 51, 21, 18, 2, 46, 22, 5, 6, 8, 0, 13)_{\Theta(1,6,6)},$
 $(0, 5, 21, 38, 34, 63, 45, 6, 48, 36, 23, 27)_{\Theta(2,2,9)},$
 $(5, 31, 42, 12, 13, 7, 4, 2, 1, 10, 24, 48)_{\Theta(2,2,9)},$
 $(0, 4, 13, 37, 23, 59, 30, 61, 54, 62, 15, 16)_{\Theta(2,3,8)},$
 $(57, 11, 55, 9, 35, 30, 19, 22, 0, 4, 20, 43)_{\Theta(2,3,8)},$
 $(0, 45, 54, 46, 12, 51, 17, 9, 58, 61, 10, 57)_{\Theta(2,4,7)},$
 $(13, 53, 40, 57, 29, 31, 6, 2, 3, 26, 50, 17)_{\Theta(2,4,7)},$
 $(0, 43, 37, 34, 58, 46, 25, 19, 41, 18, 35, 32)_{\Theta(2,5,6)},$
 $(17, 9, 25, 24, 28, 57, 48, 3, 1, 2, 15, 42)_{\Theta(2,5,6)},$
 $(0, 13, 49, 21, 52, 14, 31, 6, 35, 2, 9, 32)_{\Theta(3,4,6)},$
 $(20, 8, 19, 5, 29, 60, 62, 16, 28, 4, 25, 47)_{\Theta(3,4,6)},$
 $(0, 9, 46, 45, 1, 39, 3, 27, 38, 6, 62, 25)_{\Theta(4,4,5)},$
 $(16, 23, 33, 31, 17, 19, 7, 0, 5, 1, 14, 45)_{\Theta(4,4,5)}$

under the action of the mapping $x \mapsto x + 1 \pmod{65}$. □

E Theta graphs of 14 edges

Proof of Lemma 3.9

K_{21} Let the vertex set be $Z_{20} \cup \{\infty\}$. The decompositions consist of the graphs

$(0, 1, 2, 3, 4, 8, 5, 7, 6, 9, 13, 11, 14)_{\Theta(1,2,11)},$
 $(0, 5, 10, 6, 2, 4, 11, 1, 7, 3, 8, 16, \infty)_{\Theta(1,2,11)},$
 $(2, 10, 15, 8, 1, 9, 0, 11, 3, 14, 5, 19, \infty)_{\Theta(1,2,11)},$
 $(0, 1, 2, 3, 4, 7, 8, 5, 6, 9, 13, 11, 14)_{\Theta(1,3,10)},$
 $(0, 5, 6, 10, 7, 1, 8, 2, 4, 12, 3, 11, \infty)_{\Theta(1,3,10)},$
 $(1, 9, 10, 0, 11, 2, 7, 12, \infty, 14, 6, 19, 3)_{\Theta(1,3,10)},$
 $(0, 1, 2, 3, 4, 5, 6, 8, 11, 7, 9, 13, 10)_{\Theta(1,4,9)},$
 $(0, 4, 6, 1, 11, 8, 2, 7, 5, 12, 3, 9, \infty)_{\Theta(1,4,9)},$
 $(0, 9, 10, 3, 17, 15, 6, 14, 11, 19, \infty, 18, 2)_{\Theta(1,4,9)},$
 $(0, 1, 2, 3, 4, 7, 5, 6, 8, 12, 9, 11, 13)_{\Theta(1,5,8)},$
 $(0, 6, 7, 2, 5, 1, 8, 3, 9, 16, 10, 17, \infty)_{\Theta(1,5,8)},$
 $(1, 10, 11, 0, \infty, 3, 12, 2, 18, 6, 15, 19, 7)_{\Theta(1,5,8)},$
 $(0, 1, 2, 3, 4, 7, 5, 6, 8, 12, 9, 10, 13)_{\Theta(1,6,7)},$
 $(0, 5, 7, 1, 6, 2, 12, 8, 3, 9, 18, 4, \infty)_{\Theta(1,6,7)},$
 $(3, 7, 5, 15, 2, 10, 19, 12, 1, 14, 11, 6, \infty)_{\Theta(1,6,7)},$
 $(0, 1, 2, 3, 4, 5, 8, 6, 7, 9, 13, 10, 14)_{\Theta(2,2,10)},$
 $(0, 1, 6, 7, 5, 12, 2, 8, 3, 4, 13, \infty, 10)_{\Theta(2,2,10)},$
 $(3, 7, 10, 11, 12, 0, \infty, 15, 6, 14, 19, 5, 17)_{\Theta(2,2,10)},$
 $(0, 1, 2, 3, 4, 5, 7, 6, 8, 9, 13, 10, 14)_{\Theta(2,3,9)},$

$(0, 1, 6, 4, 11, 8, 2, 7, 3, 5, 12, \infty, 9)_{\Theta(2,3,9)},$
 $(2, 6, 15, 12, 3, 10, 1, 7, 13, 4, 19, 11, \infty)_{\Theta(2,3,9)},$
 $(0, 1, 2, 3, 4, 5, 6, 7, 9, 11, 14, 10, 12)_{\Theta(2,4,8)},$
 $(0, 1, 4, 5, 2, 7, 8, 3, 9, 14, 6, 12, \infty)_{\Theta(2,4,8)},$
 $(2, 6, 13, 11, 3, \infty, 15, 19, 12, 5, 17, 7, 16)_{\Theta(2,4,8)},$
 $(0, 1, 2, 3, 4, 5, 7, 6, 8, 12, 9, 13, 10)_{\Theta(2,5,7)},$
 $(0, 1, 8, 5, 3, 2, 6, 7, 10, 15, 4, 14, \infty)_{\Theta(2,5,7)},$
 $(3, 7, 11, 8, 2, 9, 17, 14, 6, 19, 5, 16, \infty)_{\Theta(2,5,7)},$
 $(0, 1, 2, 3, 4, 5, 7, 6, 8, 12, 9, 13, 10)_{\Theta(2,6,6)},$
 $(0, 1, 7, 5, 3, 6, 2, 8, 9, 17, 10, 4, \infty)_{\Theta(2,6,6)},$
 $(3, 7, 11, 8, 6, 15, 4, 14, 9, 19, \infty, 10, 2)_{\Theta(2,6,6)},$
 $(0, 1, 2, 3, 4, 5, 6, 8, 7, 9, 10, 13, 16)_{\Theta(3,3,8)},$
 $(0, 1, 3, 6, 7, 11, 8, 2, 9, 15, 4, 14, \infty)_{\Theta(3,3,8)},$
 $(3, 7, 8, \infty, 10, 2, 11, 17, 4, 13, 5, 14, 18)_{\Theta(3,3,8)},$
 $(0, 1, 2, 3, 4, 5, 6, 7, 8, 11, 13, 9, 12)_{\Theta(3,4,7)},$
 $(0, 1, 5, 11, 6, 2, 8, 10, 3, 7, 12, 4, \infty)_{\Theta(3,4,7)},$
 $(2, 6, 4, 15, 5, 17, 11, 9, 18, 10, 7, 19, \infty)_{\Theta(3,4,7)},$
 $(0, 1, 2, 3, 4, 5, 6, 8, 7, 9, 12, 11, 14)_{\Theta(3,5,6)},$
 $(0, 1, 3, 7, 5, 2, 6, 11, 8, 14, 4, 15, \infty)_{\Theta(3,5,6)},$
 $(2, 6, 8, 17, 10, 19, 4, \infty, 15, 3, 9, 13, 1)_{\Theta(3,5,6)},$
 $(0, 1, 2, 3, 4, 5, 6, 8, 7, 9, 11, 14, 10)_{\Theta(4,4,6)},$
 $(0, 1, 3, 7, 12, 4, 5, 9, 6, 11, 2, 8, \infty)_{\Theta(4,4,6)},$
 $(1, 5, 6, \infty, 11, 14, 2, 15, 18, 8, 16, 7, 19)_{\Theta(4,4,6)},$
 $(0, 1, 2, 3, 4, 5, 6, 8, 9, 7, 10, 13, 11)_{\Theta(4,5,5)},$
 $(0, 1, 3, 7, 5, 4, 10, 2, 6, 8, 17, 11, \infty)_{\Theta(4,5,5)},$
 $(0, 4, 10, 3, 17, 11, 2, 7, 19, 14, 5, 18, \infty)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 4 \pmod{20}$, $\infty \mapsto \infty$.

K_{28} Let the vertex set be $Z_{27} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 0, 1, 2, 3, 5, 4, 6, 9, 13, 7, 10, 8)_{\Theta(1,2,11)},$
 $(0, 5, 9, 6, 1, 8, 2, 7, 11, 14, 3, 10, 18)_{\Theta(1,2,11)},$
 $(0, 10, 20, 12, 1, 13, 2, 11, 21, 7, 16, 8, 23)_{\Theta(1,2,11)},$
 $(\infty, 0, 1, 2, 5, 6, 3, 4, 7, 9, 13, 8, 10)_{\Theta(1,3,10)},$
 $(0, 5, 6, 1, 7, 13, 2, 8, 11, 3, 12, 4, 14)_{\Theta(1,3,10)},$
 $(0, 13, 11, 4, 12, 1, 14, 2, 6, 23, 3, 17, 25)_{\Theta(1,3,10)},$
 $(\infty, 0, 1, 2, 3, 5, 7, 4, 6, 8, 11, 15, 9)_{\Theta(1,4,9)},$
 $(0, 1, 4, 8, 13, 5, 11, 2, 9, 16, 3, 14, 6)_{\Theta(1,4,9)},$
 $(0, 14, 10, 1, 7, 12, 2, 13, 23, 11, 25, 6, 22)_{\Theta(1,4,9)},$
 $(\infty, 0, 1, 2, 3, 4, 5, 7, 9, 6, 8, 11, 15)_{\Theta(1,5,8)},$
 $(0, 5, 6, 1, 4, 10, 7, 11, 2, 8, 15, 23, 13)_{\Theta(1,5,8)},$
 $(0, 10, 9, 1, 8, 19, 13, 26, 12, 2, 17, 6, 22)_{\Theta(1,5,8)},$
 $(\infty, 0, 1, 2, 3, 4, 6, 5, 7, 10, 14, 8, 11)_{\Theta(1,6,7)},$
 $(0, 2, 3, 7, 1, 6, 11, 8, 12, 4, 13, 5, 15)_{\Theta(1,6,7)},$
 $(0, 7, 9, 2, 13, 1, 18, 12, 25, 11, 16, 26, 14)_{\Theta(1,6,7)},$

$(\infty, 0, 1, 2, 3, 6, 4, 5, 7, 10, 14, 8, 9)_{\Theta(2,2,10)},$
 $(0, 1, 6, 7, 4, 11, 2, 5, 9, 14, 3, 13, 23)_{\Theta(2,2,10)},$
 $(0, 2, 12, 13, 8, 15, 4, 16, 7, 20, 5, 18, 10)_{\Theta(2,2,10)},$
 $(\infty, 0, 1, 2, 3, 6, 4, 5, 7, 10, 14, 8, 11)_{\Theta(2,3,9)},$
 $(0, 1, 6, 2, 7, 4, 11, 3, 8, 12, 5, 13, 21)_{\Theta(2,3,9)},$
 $(0, 2, 12, 9, 19, 13, 1, 10, 21, 8, 17, 5, 16)_{\Theta(2,3,9)},$
 $(\infty, 0, 1, 2, 3, 5, 6, 4, 7, 8, 10, 14, 11)_{\Theta(2,4,8)},$
 $(0, 1, 6, 3, 7, 13, 8, 2, 9, 5, 10, 19, 11)_{\Theta(2,4,8)},$
 $(0, 2, 12, 7, 20, 13, 9, 1, 15, 4, 21, 8, 17)_{\Theta(2,4,8)},$
 $(\infty, 0, 1, 2, 3, 5, 4, 6, 9, 7, 10, 8, 11)_{\Theta(2,5,7)},$
 $(0, 1, 5, 6, 2, 7, 12, 8, 14, 4, 10, 3, 13)_{\Theta(2,5,7)},$
 $(0, 2, 9, 12, 4, 17, 10, 13, 22, 11, 20, 5, 15)_{\Theta(2,5,7)},$
 $(\infty, 0, 1, 2, 3, 5, 4, 6, 9, 12, 7, 10, 8)_{\Theta(2,6,6)},$
 $(0, 1, 5, 4, 10, 2, 6, 13, 9, 19, 3, 14, 8)_{\Theta(2,6,6)},$
 $(0, 2, 12, 13, 4, 14, 11, 16, 20, 5, 23, 7, 15)_{\Theta(2,6,6)},$
 $(\infty, 0, 1, 2, 3, 4, 5, 6, 9, 7, 10, 8, 11)_{\Theta(3,3,8)},$
 $(0, 1, 5, 9, 6, 13, 8, 2, 7, 11, 4, 10, 15)_{\Theta(3,3,8)},$
 $(0, 2, 10, 19, 18, 11, 12, 1, 14, 25, 17, 5, 15)_{\Theta(3,3,8)},$
 $(\infty, 0, 1, 2, 3, 4, 6, 5, 7, 10, 14, 8, 9)_{\Theta(3,4,7)},$
 $(0, 1, 3, 7, 5, 2, 6, 8, 13, 4, 11, 19, 9)_{\Theta(3,4,7)},$
 $(0, 2, 7, 17, 11, 21, 9, 13, 26, 8, 19, 4, 15)_{\Theta(3,4,7)},$
 $(\infty, 0, 1, 2, 3, 4, 6, 5, 8, 10, 7, 11, 14)_{\Theta(3,5,6)},$
 $(0, 1, 3, 7, 6, 2, 8, 13, 9, 4, 11, 18, 10)_{\Theta(3,5,6)},$
 $(0, 2, 7, 17, 8, 16, 3, 13, 12, 23, 10, 21, 11)_{\Theta(3,5,6)},$
 $(\infty, 0, 1, 2, 3, 5, 7, 4, 6, 8, 11, 15, 9)_{\Theta(4,4,6)},$
 $(0, 1, 5, 10, 3, 8, 2, 9, 11, 4, 13, 7, 6)_{\Theta(4,4,6)},$
 $(0, 2, 10, 20, 16, 12, 1, 13, 14, 22, 9, 26, 11)_{\Theta(4,4,6)},$
 $(\infty, 0, 1, 2, 3, 5, 7, 4, 6, 9, 10, 14, 8)_{\Theta(4,5,5)},$
 $(0, 1, 2, 5, 9, 4, 10, 3, 8, 11, 16, 6, 15)_{\Theta(4,5,5)},$
 $(0, 2, 12, 1, 10, 17, 3, 23, 11, 22, 8, 25, 13)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 3 \pmod{27}$, $\infty \mapsto \infty$.

K_{36} Let the vertex set be Z_{36} . The decompositions consist of

$(21, 2, 19, 3, 9, 1, 23, 7, 29, 16, 24, 15, 27)_{\Theta(1,2,11)},$
 $(24, 26, 6, 21, 14, 28, 9, 18, 15, 10, 2, 23, 0)_{\Theta(1,2,11)},$
 $(3, 7, 33, 4, 8, 17, 2, 32, 6, 1, 26, 19, 0)_{\Theta(1,2,11)},$
 $(31, 29, 30, 22, 18, 24, 27, 16, 1, 5, 12, 28, 6)_{\Theta(1,2,11)},$
 $(1, 34, 0, 12, 7, 17, 29, 9, 4, 16, 31, 23, 10)_{\Theta(1,2,11)},$
 $(0, 23, 30, 7, 21, 6, 10, 26, 9, 8, 25, 15, 13)_{\Theta(1,3,10)},$
 $(30, 27, 2, 12, 1, 34, 3, 5, 32, 4, 20, 31, 6)_{\Theta(1,3,10)},$
 $(2, 19, 13, 33, 4, 8, 14, 26, 27, 31, 22, 20, 25)_{\Theta(1,3,10)},$
 $(33, 0, 2, 29, 3, 31, 19, 1, 5, 18, 4, 26, 25)_{\Theta(1,3,10)},$
 $(2, 20, 28, 11, 31, 0, 3, 4, 16, 29, 1, 13, 27)_{\Theta(1,3,10)},$
 $(0, 1, 30, 5, 2, 21, 14, 19, 22, 13, 33, 4, 24)_{\Theta(1,4,9)},$
 $(3, 20, 9, 13, 1, 27, 14, 26, 30, 32, 34, 24, 10)_{\Theta(1,4,9)},$

$(23, 0, 25, 3, 24, 17, 27, 9, 12, 21, 29, 6, 7)_{\Theta(1,4,9)},$
 $(3, 12, 13, 24, 23, 14, 34, 29, 10, 27, 25, 11, 16)_{\Theta(1,4,9)},$
 $(12, 18, 4, 7, 3, 17, 2, 10, 28, 6, 35, 19, 27)_{\Theta(1,4,9)},$
 $(0, 19, 9, 6, 18, 14, 16, 22, 33, 29, 10, 5, 34)_{\Theta(1,5,8)},$
 $(26, 28, 27, 21, 33, 19, 17, 0, 11, 34, 31, 22, 2)_{\Theta(1,5,8)},$
 $(34, 12, 23, 28, 17, 24, 16, 15, 27, 31, 21, 0, 8)_{\Theta(1,5,8)},$
 $(24, 21, 18, 17, 31, 1, 25, 12, 14, 7, 27, 4, 19)_{\Theta(1,5,8)},$
 $(6, 16, 14, 0, 3, 21, 29, 31, 5, 13, 34, 15, 23)_{\Theta(1,5,8)},$
 $(0, 9, 7, 26, 1, 16, 6, 15, 3, 5, 27, 34, 30)_{\Theta(1,6,7)},$
 $(15, 6, 21, 26, 8, 19, 11, 12, 29, 13, 1, 27, 14)_{\Theta(1,6,7)},$
 $(0, 30, 34, 18, 3, 4, 16, 23, 13, 27, 9, 26, 20)_{\Theta(1,6,7)},$
 $(22, 9, 19, 23, 7, 24, 20, 34, 32, 25, 17, 10, 11)_{\Theta(1,6,7)},$
 $(16, 13, 0, 1, 5, 11, 22, 7, 12, 20, 33, 28, 14)_{\Theta(1,6,7)},$
 $(0, 13, 17, 33, 8, 28, 32, 25, 19, 4, 9, 26, 2)_{\Theta(2,2,10)},$
 $(21, 13, 6, 3, 30, 15, 28, 34, 1, 11, 27, 16, 19)_{\Theta(2,2,10)},$
 $(30, 26, 32, 27, 12, 14, 18, 11, 3, 5, 20, 29, 34)_{\Theta(2,2,10)},$
 $(22, 2, 12, 27, 31, 32, 20, 6, 26, 3, 7, 24, 19)_{\Theta(2,2,10)},$
 $(15, 2, 1, 24, 3, 17, 4, 5, 16, 23, 21, 13, 25)_{\Theta(2,2,10)},$
 $(0, 24, 17, 19, 22, 28, 13, 25, 14, 29, 33, 6, 30)_{\Theta(2,3,9)},$
 $(17, 15, 10, 19, 3, 7, 18, 1, 29, 32, 6, 2, 12)_{\Theta(2,3,9)},$
 $(29, 3, 16, 7, 21, 34, 4, 26, 24, 33, 8, 13, 11)_{\Theta(2,3,9)},$
 $(30, 4, 11, 22, 15, 3, 8, 24, 28, 6, 9, 25, 19)_{\Theta(2,3,9)},$
 $(25, 3, 2, 24, 12, 26, 6, 27, 14, 32, 31, 35, 29)_{\Theta(2,3,9)},$
 $(0, 30, 14, 5, 20, 3, 22, 10, 25, 7, 6, 29, 19)_{\Theta(2,4,8)},$
 $(34, 3, 33, 2, 20, 26, 5, 10, 15, 17, 9, 19, 31)_{\Theta(2,4,8)},$
 $(31, 10, 0, 15, 24, 12, 14, 5, 27, 23, 30, 2, 13)_{\Theta(2,4,8)},$
 $(19, 3, 16, 12, 5, 1, 8, 25, 6, 27, 33, 13, 4)_{\Theta(2,4,8)},$
 $(11, 9, 32, 14, 12, 8, 33, 21, 24, 18, 28, 0, 20)_{\Theta(2,4,8)},$
 $(0, 19, 16, 5, 32, 29, 13, 22, 24, 17, 18, 7, 33)_{\Theta(2,5,7)},$
 $(34, 28, 2, 27, 12, 20, 3, 9, 33, 14, 6, 21, 32)_{\Theta(2,5,7)},$
 $(11, 22, 2, 26, 32, 14, 27, 10, 8, 31, 17, 19, 29)_{\Theta(2,5,7)},$
 $(24, 17, 14, 9, 18, 6, 0, 15, 20, 21, 13, 11, 4)_{\Theta(2,5,7)},$
 $(15, 2, 19, 3, 6, 29, 33, 7, 13, 31, 11, 12, 24)_{\Theta(2,5,7)},$
 $(0, 2, 25, 28, 1, 21, 20, 18, 10, 17, 26, 15, 6)_{\Theta(2,6,6)},$
 $(9, 28, 7, 3, 29, 15, 23, 5, 34, 1, 13, 16, 30)_{\Theta(2,6,6)},$
 $(28, 20, 3, 15, 16, 11, 7, 1, 10, 18, 33, 14, 8)_{\Theta(2,6,6)},$
 $(7, 28, 22, 31, 14, 13, 5, 19, 4, 30, 18, 23, 33)_{\Theta(2,6,6)},$
 $(20, 0, 4, 5, 1, 3, 6, 7, 13, 18, 31, 15, 22)_{\Theta(2,6,6)},$
 $(0, 7, 24, 8, 21, 5, 28, 11, 17, 9, 26, 13, 25)_{\Theta(3,3,8)},$
 $(32, 25, 26, 30, 21, 14, 22, 16, 7, 23, 1, 24, 10)_{\Theta(3,3,8)},$
 $(34, 31, 26, 2, 14, 19, 1, 10, 12, 8, 11, 3, 33)_{\Theta(3,3,8)},$
 $(34, 6, 23, 27, 15, 5, 7, 20, 21, 12, 17, 31, 24)_{\Theta(3,3,8)},$
 $(11, 0, 14, 15, 16, 33, 21, 17, 24, 26, 12, 23, 10)_{\Theta(3,3,8)},$
 $(0, 19, 24, 9, 8, 2, 13, 10, 34, 7, 25, 22, 23)_{\Theta(3,4,7)},$
 $(9, 23, 25, 15, 3, 34, 2, 7, 28, 8, 13, 12, 30)_{\Theta(3,4,7)},$

$(28, 20, 5, 22, 27, 14, 3, 18, 34, 21, 23, 32, 13)_{\Theta(3,4,7)},$
 $(29, 9, 33, 2, 14, 22, 0, 21, 32, 28, 15, 1, 10)_{\Theta(3,4,7)},$
 $(34, 3, 0, 6, 15, 27, 32, 20, 23, 12, 9, 33, 19)_{\Theta(3,4,7)},$
 $(0, 31, 27, 17, 15, 25, 2, 12, 29, 1, 6, 4, 7)_{\Theta(3,5,6)},$
 $(13, 5, 7, 6, 1, 18, 31, 27, 9, 28, 23, 24, 2)_{\Theta(3,5,6)},$
 $(1, 9, 32, 18, 26, 11, 19, 30, 24, 8, 14, 22, 2)_{\Theta(3,5,6)},$
 $(7, 1, 27, 21, 5, 16, 20, 28, 0, 30, 18, 22, 4)_{\Theta(3,5,6)},$
 $(14, 3, 4, 5, 7, 10, 15, 28, 31, 22, 24, 0, 21)_{\Theta(3,5,6)},$
 $(0, 2, 8, 30, 17, 7, 32, 22, 1, 15, 5, 11, 27)_{\Theta(4,4,6)},$
 $(15, 7, 29, 19, 6, 17, 34, 16, 3, 0, 33, 24, 12)_{\Theta(4,4,6)},$
 $(2, 12, 1, 20, 14, 26, 30, 33, 23, 21, 8, 4, 18)_{\Theta(4,4,6)},$
 $(8, 7, 13, 31, 3, 33, 21, 14, 6, 1, 29, 23, 24)_{\Theta(4,4,6)},$
 $(17, 2, 1, 5, 30, 24, 4, 19, 26, 16, 3, 6, 11)_{\Theta(4,4,6)},$
 $(0, 22, 19, 33, 7, 31, 32, 17, 11, 27, 4, 28, 1)_{\Theta(4,5,5)},$
 $(6, 25, 7, 19, 12, 13, 15, 33, 32, 17, 28, 10, 9)_{\Theta(4,5,5)},$
 $(10, 17, 22, 28, 31, 33, 2, 34, 7, 12, 23, 26, 20)_{\Theta(4,5,5)},$
 $(12, 15, 2, 30, 11, 26, 29, 10, 0, 16, 24, 4, 9)_{\Theta(4,5,5)},$
 $(0, 5, 17, 21, 29, 2, 7, 30, 14, 22, 15, 23, 3)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 4 \pmod{36}$.

K_{49} Let the vertex set be Z_{49} . The decompositions consist of

$(42, 34, 3, 38, 41, 5, 27, 14, 19, 4, 2, 35, 20)_{\Theta(1,2,11)},$
 $(47, 13, 37, 4, 8, 0, 32, 9, 28, 34, 25, 26, 35)_{\Theta(1,2,11)},$
 $(24, 36, 23, 47, 43, 20, 30, 5, 15, 27, 33, 4, 0)_{\Theta(1,2,11)},$
 $(13, 14, 34, 25, 24, 17, 33, 45, 47, 30, 28, 7, 36)_{\Theta(1,2,11)},$
 $(38, 35, 23, 30, 33, 47, 11, 36, 6, 25, 28, 37, 17)_{\Theta(1,2,11)},$
 $(9, 6, 8, 39, 24, 11, 16, 3, 27, 17, 25, 47, 22)_{\Theta(1,2,11)},$
 $(40, 22, 29, 43, 24, 8, 46, 39, 18, 32, 6, 5, 7)_{\Theta(1,2,11)},$
 $(21, 10, 39, 35, 5, 47, 15, 17, 3, 29, 32, 27, 38)_{\Theta(1,2,11)},$
 $(38, 6, 44, 34, 30, 19, 10, 37, 9, 1, 23, 28, 4)_{\Theta(1,2,11)},$
 $(34, 29, 43, 18, 9, 5, 46, 24, 7, 19, 40, 45, 39)_{\Theta(1,2,11)},$
 $(16, 32, 42, 23, 41, 34, 5, 28, 35, 10, 40, 7, 1)_{\Theta(1,2,11)},$
 $(1, 18, 30, 10, 15, 9, 40, 48, 16, 2, 36, 8, 7)_{\Theta(1,2,11)},$
 $(0, 25, 33, 29, 26, 5, 35, 22, 17, 39, 19, 20, 1)_{\Theta(1,3,10)},$
 $(35, 31, 40, 37, 7, 9, 34, 43, 20, 30, 25, 23, 5)_{\Theta(1,3,10)},$
 $(30, 19, 27, 3, 21, 31, 43, 14, 44, 11, 24, 15, 23)_{\Theta(1,3,10)},$
 $(7, 45, 46, 16, 18, 35, 9, 20, 2, 11, 19, 28, 44)_{\Theta(1,3,10)},$
 $(33, 46, 31, 38, 22, 21, 35, 47, 15, 4, 39, 12, 34)_{\Theta(1,3,10)},$
 $(29, 8, 11, 43, 32, 6, 2, 12, 17, 15, 16, 21, 18)_{\Theta(1,3,10)},$
 $(22, 45, 3, 41, 14, 13, 34, 40, 10, 7, 5, 4, 24)_{\Theta(1,3,10)},$
 $(29, 19, 41, 43, 36, 42, 24, 33, 13, 21, 39, 45, 31)_{\Theta(1,3,10)},$
 $(47, 29, 32, 13, 1, 28, 45, 6, 24, 8, 30, 16, 44)_{\Theta(1,3,10)},$
 $(30, 38, 3, 6, 13, 26, 32, 4, 20, 18, 44, 8, 2)_{\Theta(1,3,10)},$
 $(36, 21, 41, 34, 16, 9, 24, 25, 18, 30, 5, 40, 6)_{\Theta(1,3,10)},$
 $(7, 0, 11, 6, 27, 41, 32, 17, 42, 20, 12, 5, 37)_{\Theta(1,3,10)},$
 $(0, 45, 38, 35, 23, 2, 32, 4, 22, 16, 6, 47, 3)_{\Theta(1,4,9)},$
 $(47, 1, 17, 32, 19, 39, 3, 6, 25, 5, 40, 44, 22)_{\Theta(1,4,9)},$

$(21, 28, 1, 46, 3, 27, 13, 17, 29, 6, 42, 10, 44)_{\Theta(1,4,9)},$
 $(10, 27, 24, 3, 2, 47, 16, 21, 39, 35, 14, 22, 29)_{\Theta(1,4,9)},$
 $(19, 43, 9, 1, 17, 15, 30, 2, 31, 47, 45, 6, 34)_{\Theta(1,4,9)},$
 $(13, 40, 1, 10, 33, 4, 2, 25, 41, 26, 27, 0, 20)_{\Theta(1,4,9)},$
 $(35, 21, 20, 16, 36, 8, 7, 17, 25, 27, 30, 43, 26)_{\Theta(1,4,9)},$
 $(40, 19, 18, 45, 25, 21, 20, 15, 10, 41, 46, 14, 6)_{\Theta(1,4,9)},$
 $(20, 13, 26, 37, 44, 45, 5, 21, 19, 16, 3, 1, 39)_{\Theta(1,4,9)},$
 $(40, 25, 1, 18, 9, 14, 23, 0, 11, 12, 37, 2, 13)_{\Theta(1,4,9)},$
 $(9, 46, 4, 29, 32, 26, 15, 28, 18, 17, 23, 6, 36)_{\Theta(1,4,9)},$
 $(23, 42, 31, 1, 36, 22, 6, 17, 35, 11, 18, 21, 33)_{\Theta(1,4,9)},$
 $(0, 1, 43, 6, 39, 16, 47, 17, 14, 35, 32, 18, 31)_{\Theta(1,5,8)},$
 $(43, 21, 27, 25, 23, 47, 12, 36, 29, 19, 10, 28, 39)_{\Theta(1,5,8)},$
 $(22, 11, 41, 26, 39, 2, 14, 34, 0, 32, 17, 3, 15)_{\Theta(1,5,8)},$
 $(41, 9, 1, 27, 5, 3, 42, 29, 24, 15, 43, 10, 20)_{\Theta(1,5,8)},$
 $(13, 38, 31, 26, 18, 10, 19, 2, 14, 29, 43, 45, 30)_{\Theta(1,5,8)},$
 $(12, 18, 2, 0, 41, 37, 42, 33, 32, 4, 31, 34, 17)_{\Theta(1,5,8)},$
 $(2, 38, 21, 5, 1, 32, 15, 44, 26, 11, 8, 19, 45)_{\Theta(1,5,8)},$
 $(11, 20, 35, 30, 27, 22, 36, 46, 17, 33, 1, 2, 44)_{\Theta(1,5,8)},$
 $(43, 37, 14, 18, 28, 40, 17, 16, 45, 35, 44, 34, 4)_{\Theta(1,5,8)},$
 $(35, 0, 28, 45, 7, 13, 10, 14, 30, 8, 25, 18, 23)_{\Theta(1,5,8)},$
 $(9, 47, 23, 2, 43, 40, 5, 34, 11, 6, 41, 45, 33)_{\Theta(1,5,8)},$
 $(32, 20, 5, 6, 28, 33, 12, 40, 48, 30, 3, 41, 13)_{\Theta(1,5,8)},$
 $(0, 17, 28, 34, 8, 43, 14, 19, 30, 35, 24, 5, 36)_{\Theta(1,6,7)},$
 $(21, 20, 39, 10, 9, 28, 2, 45, 11, 38, 42, 26, 41)_{\Theta(1,6,7)},$
 $(30, 23, 27, 38, 17, 15, 18, 39, 19, 25, 37, 10, 36)_{\Theta(1,6,7)},$
 $(28, 22, 5, 27, 11, 24, 44, 26, 38, 3, 42, 2, 30)_{\Theta(1,6,7)},$
 $(4, 14, 36, 0, 2, 16, 46, 18, 42, 20, 38, 47, 11)_{\Theta(1,6,7)},$
 $(45, 13, 38, 39, 46, 22, 10, 4, 37, 19, 29, 24, 15)_{\Theta(1,6,7)},$
 $(14, 26, 34, 4, 19, 22, 6, 18, 8, 0, 42, 27, 15)_{\Theta(1,6,7)},$
 $(6, 11, 5, 37, 47, 45, 9, 15, 4, 25, 31, 26, 20)_{\Theta(1,6,7)},$
 $(19, 6, 3, 13, 43, 15, 14, 28, 10, 33, 12, 39, 41)_{\Theta(1,6,7)},$
 $(35, 23, 21, 36, 3, 44, 27, 46, 34, 26, 33, 25, 29)_{\Theta(1,6,7)},$
 $(33, 8, 32, 9, 24, 30, 15, 19, 16, 41, 34, 29, 12)_{\Theta(1,6,7)},$
 $(40, 16, 35, 2, 22, 4, 27, 44, 34, 10, 6, 42, 15)_{\Theta(1,6,7)},$
 $(0, 30, 28, 24, 42, 7, 13, 29, 6, 45, 39, 44, 10)_{\Theta(2,3,9)},$
 $(25, 18, 9, 40, 30, 3, 26, 31, 8, 4, 34, 21, 1)_{\Theta(2,3,9)},$
 $(46, 18, 34, 44, 43, 17, 20, 29, 47, 12, 41, 42, 37)_{\Theta(2,3,9)},$
 $(35, 18, 12, 20, 11, 24, 43, 32, 0, 4, 3, 9, 15)_{\Theta(2,3,9)},$
 $(13, 25, 11, 42, 43, 37, 15, 44, 12, 14, 31, 19, 47)_{\Theta(2,3,9)},$
 $(46, 42, 26, 20, 23, 28, 25, 38, 10, 18, 3, 27, 9)_{\Theta(2,3,9)},$
 $(43, 28, 15, 9, 22, 29, 21, 44, 45, 35, 5, 4, 42)_{\Theta(2,3,9)},$
 $(5, 19, 27, 12, 29, 23, 14, 6, 8, 18, 13, 26, 30)_{\Theta(2,3,9)},$
 $(24, 18, 26, 21, 46, 33, 36, 38, 19, 15, 8, 27, 44)_{\Theta(2,3,9)},$
 $(37, 45, 23, 41, 3, 9, 20, 13, 19, 28, 33, 34, 29)_{\Theta(2,3,9)},$
 $(33, 5, 30, 8, 42, 22, 34, 20, 41, 45, 36, 24, 38)_{\Theta(2,3,9)},$
 $(6, 1, 28, 16, 9, 38, 2, 10, 41, 26, 39, 0, 45)_{\Theta(2,3,9)},$
 $(0, 43, 44, 45, 2, 29, 42, 37, 3, 14, 13, 36, 12)_{\Theta(2,5,7)},$

$(31, 43, 22, 39, 28, 47, 8, 9, 30, 23, 34, 41, 2)_{\Theta(2,5,7)},$
 $(26, 27, 11, 6, 8, 20, 45, 13, 5, 31, 23, 39, 0)_{\Theta(2,5,7)},$
 $(44, 1, 45, 29, 40, 43, 11, 32, 8, 46, 34, 9, 35)_{\Theta(2,5,7)},$
 $(3, 31, 15, 35, 8, 39, 7, 41, 23, 12, 2, 37, 1)_{\Theta(2,5,7)},$
 $(27, 0, 14, 13, 16, 36, 43, 21, 18, 32, 31, 41, 8)_{\Theta(2,5,7)},$
 $(35, 2, 19, 16, 47, 18, 25, 27, 42, 40, 36, 30, 34)_{\Theta(2,5,7)},$
 $(42, 2, 35, 46, 25, 26, 38, 3, 1, 4, 31, 33, 47)_{\Theta(2,5,7)},$
 $(33, 44, 42, 30, 25, 20, 46, 7, 19, 40, 8, 31, 24)_{\Theta(2,5,7)},$
 $(18, 32, 38, 42, 45, 14, 23, 37, 12, 13, 47, 25, 34)_{\Theta(2,5,7)},$
 $(1, 17, 20, 21, 26, 18, 31, 41, 24, 3, 19, 32, 13)_{\Theta(2,5,7)},$
 $(40, 5, 10, 13, 8, 4, 47, 31, 46, 28, 0, 20, 48)_{\Theta(2,5,7)},$
 $(0, 20, 28, 34, 13, 5, 18, 37, 33, 6, 25, 32, 4)_{\Theta(3,3,8)},$
 $(43, 33, 2, 15, 32, 38, 6, 46, 29, 36, 28, 24, 10)_{\Theta(3,3,8)},$
 $(42, 10, 5, 43, 19, 6, 35, 36, 1, 45, 23, 13, 34)_{\Theta(3,3,8)},$
 $(29, 5, 26, 1, 20, 44, 4, 39, 23, 38, 28, 10, 46)_{\Theta(3,3,8)},$
 $(3, 18, 6, 21, 5, 30, 44, 39, 10, 36, 46, 19, 28)_{\Theta(3,3,8)},$
 $(28, 13, 9, 30, 6, 8, 43, 27, 32, 7, 12, 41, 31)_{\Theta(3,3,8)},$
 $(12, 46, 31, 20, 32, 1, 13, 2, 0, 23, 7, 42, 34)_{\Theta(3,3,8)},$
 $(0, 23, 11, 37, 32, 17, 36, 38, 39, 41, 21, 24, 43)_{\Theta(3,3,8)},$
 $(28, 38, 26, 5, 16, 47, 27, 1, 29, 12, 18, 33, 14)_{\Theta(3,3,8)},$
 $(37, 7, 6, 2, 22, 16, 24, 41, 34, 15, 42, 26, 36)_{\Theta(3,3,8)},$
 $(37, 38, 5, 17, 30, 29, 34, 40, 26, 33, 46, 43, 0)_{\Theta(3,3,8)},$
 $(26, 44, 23, 24, 37, 46, 25, 17, 39, 35, 3, 10, 22)_{\Theta(3,3,8)},$
 $(0, 45, 7, 12, 8, 2, 41, 22, 47, 35, 21, 39, 16)_{\Theta(3,4,7)},$
 $(32, 2, 7, 35, 12, 6, 24, 46, 30, 16, 40, 47, 19)_{\Theta(3,4,7)},$
 $(44, 47, 4, 7, 42, 3, 1, 32, 2, 27, 15, 0, 20)_{\Theta(3,4,7)},$
 $(13, 30, 32, 43, 4, 1, 40, 14, 20, 45, 10, 16, 34)_{\Theta(3,4,7)},$
 $(38, 1, 12, 25, 4, 9, 21, 43, 29, 10, 13, 33, 32)_{\Theta(3,4,7)},$
 $(14, 45, 6, 17, 37, 19, 25, 40, 29, 3, 10, 44, 33)_{\Theta(3,4,7)},$
 $(3, 14, 21, 17, 28, 45, 46, 15, 37, 44, 36, 19, 27)_{\Theta(3,4,7)},$
 $(36, 26, 34, 24, 45, 32, 11, 0, 4, 8, 37, 7, 41)_{\Theta(3,4,7)},$
 $(1, 17, 7, 34, 8, 25, 47, 5, 10, 32, 24, 23, 26)_{\Theta(3,4,7)},$
 $(38, 36, 2, 13, 32, 6, 5, 30, 29, 44, 23, 20, 27)_{\Theta(3,4,7)},$
 $(44, 20, 40, 32, 27, 13, 18, 46, 39, 29, 1, 6, 36)_{\Theta(3,4,7)},$
 $(14, 11, 9, 0, 12, 48, 27, 15, 31, 42, 26, 40, 21)_{\Theta(3,4,7)},$
 $(0, 5, 31, 14, 3, 41, 15, 44, 43, 13, 8, 40, 29)_{\Theta(3,5,6)},$
 $(35, 32, 34, 20, 19, 29, 21, 8, 14, 7, 4, 11, 45)_{\Theta(3,5,6)},$
 $(42, 33, 27, 17, 32, 18, 22, 29, 37, 23, 26, 41, 35)_{\Theta(3,5,6)},$
 $(34, 26, 44, 17, 14, 28, 5, 4, 25, 0, 22, 1, 13)_{\Theta(3,5,6)},$
 $(0, 35, 10, 16, 29, 3, 20, 9, 38, 42, 8, 11, 23)_{\Theta(3,5,6)},$
 $(7, 32, 25, 30, 20, 27, 6, 9, 12, 44, 1, 17, 31)_{\Theta(3,5,6)},$
 $(30, 2, 14, 38, 22, 23, 21, 33, 37, 9, 5, 35, 18)_{\Theta(3,5,6)},$
 $(0, 34, 1, 36, 9, 33, 47, 26, 41, 16, 5, 24, 43)_{\Theta(3,5,6)},$
 $(45, 0, 33, 4, 20, 39, 6, 11, 17, 16, 43, 31, 27)_{\Theta(3,5,6)},$
 $(4, 10, 40, 33, 44, 8, 46, 18, 22, 6, 24, 17, 39)_{\Theta(3,5,6)},$
 $(41, 4, 23, 6, 38, 32, 47, 27, 40, 22, 37, 45, 36)_{\Theta(3,5,6)},$
 $(34, 5, 30, 10, 12, 18, 37, 3, 40, 43, 38, 36, 46)_{\Theta(3,5,6)},$

$(0, 3, 15, 2, 34, 26, 27, 16, 31, 33, 39, 13, 28)_{\Theta(4,5,5)},$
 $(19, 23, 32, 27, 17, 10, 15, 4, 25, 9, 30, 47, 45)_{\Theta(4,5,5)},$
 $(9, 38, 0, 21, 22, 34, 18, 8, 32, 13, 10, 11, 7)_{\Theta(4,5,5)},$
 $(38, 23, 18, 44, 37, 21, 11, 30, 7, 2, 0, 13, 8)_{\Theta(4,5,5)},$
 $(24, 45, 23, 19, 31, 22, 28, 17, 41, 39, 36, 30, 42)_{\Theta(4,5,5)},$
 $(30, 2, 35, 0, 5, 20, 27, 19, 46, 1, 40, 39, 21)_{\Theta(4,5,5)},$
 $(3, 45, 41, 13, 40, 23, 12, 29, 15, 7, 39, 31, 8)_{\Theta(4,5,5)},$
 $(36, 4, 33, 5, 19, 14, 43, 6, 8, 37, 45, 32, 39)_{\Theta(4,5,5)},$
 $(35, 19, 5, 23, 11, 45, 36, 12, 21, 32, 15, 37, 34)_{\Theta(4,5,5)},$
 $(22, 14, 13, 29, 6, 33, 21, 28, 39, 41, 47, 18, 20)_{\Theta(4,5,5)},$
 $(3, 44, 10, 27, 13, 25, 6, 26, 19, 33, 17, 40, 36)_{\Theta(4,5,5)},$
 $(4, 0, 44, 11, 20, 22, 15, 28, 27, 41, 5, 36, 8)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 7 \pmod{49}$.

K_{56} Let the vertex set be $Z_{55} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 38, 40, 19, 10, 30, 15, 46, 42, 35, 18, 44, 32)_{\Theta(1,2,11)},$
 $(26, 27, 7, 41, 46, 1, 48, 13, 5, 50, 29, 14, 10)_{\Theta(1,2,11)},$
 $(15, 38, 20, 40, 32, 30, 43, 0, 27, 28, 4, 41, 23)_{\Theta(1,2,11)},$
 $(44, 3, 1, 52, 11, 50, 24, 47, 25, 53, 14, 46, 7)_{\Theta(1,2,11)},$
 $(12, 23, 29, 43, 48, 15, 26, 38, 35, 31, 28, 51, 24)_{\Theta(1,2,11)},$
 $(53, 26, 17, 31, 51, 25, 14, 16, 49, 3, 24, 52, 37)_{\Theta(1,2,11)},$
 $(36, 29, 19, 15, 12, 35, 47, 14, 1, 10, 4, 52, 18)_{\Theta(1,2,11)},$
 $(26, 1, 43, 52, 46, 22, 47, 29, 15, 39, 42, 21, 20)_{\Theta(1,2,11)},$
 $(12, 14, 30, 17, 27, 18, 25, 11, 5, 4, 23, 13, 9)_{\Theta(1,2,11)},$
 $(43, 13, 27, 17, 4, 1, 9, 29, 54, 35, 22, \infty, 6)_{\Theta(1,2,11)},$
 $(\infty, 26, 5, 25, 48, 13, 8, 33, 15, 45, 35, 51, 4)_{\Theta(1,3,10)},$
 $(22, 42, 53, 10, 14, 40, 49, 51, 32, 26, 16, 13, 12)_{\Theta(1,3,10)},$
 $(45, 40, 34, 43, 37, 20, 2, 42, 1, 8, 24, 22, 39)_{\Theta(1,3,10)},$
 $(50, 33, 28, 14, 6, 35, 4, 3, 30, 43, 7, 34, 29)_{\Theta(1,3,10)},$
 $(27, 38, 48, 40, 51, 33, 18, 2, 39, 13, 7, 12, 44)_{\Theta(1,3,10)},$
 $(4, 37, 10, 44, 21, 42, 13, 27, 31, 46, 34, 14, 30)_{\Theta(1,3,10)},$
 $(49, 1, 26, 43, 19, 5, 28, 16, 35, 21, 53, 7, 17)_{\Theta(1,3,10)},$
 $(51, 53, 0, 2, 20, 26, 52, 39, 8, 36, 54, 50, 43)_{\Theta(1,3,10)},$
 $(4, 14, 1, 29, 46, 21, 43, 54, 2, 5, 32, 10, 50)_{\Theta(1,3,10)},$
 $(2, 14, 1, 48, 11, 6, 15, 27, 16, 51, 30, 17, \infty)_{\Theta(1,3,10)},$
 $(\infty, 17, 35, 48, 52, 13, 33, 2, 45, 40, 18, 1, 24)_{\Theta(1,4,9)},$
 $(51, 23, 46, 25, 9, 34, 31, 18, 45, 47, 7, 54, 0)_{\Theta(1,4,9)},$
 $(31, 41, 4, 0, 30, 44, 33, 7, 23, 29, 54, 11, 47)_{\Theta(1,4,9)},$
 $(44, 47, 54, 18, 33, 53, 21, 13, 34, 38, 17, 42, 48)_{\Theta(1,4,9)},$
 $(13, 15, 38, 31, 33, 1, 27, 20, 7, 40, 2, 12, 29)_{\Theta(1,4,9)},$
 $(22, 36, 44, 0, 10, 17, 25, 31, 53, 50, 3, 15, 11)_{\Theta(1,4,9)},$
 $(19, 34, 39, 37, 10, 1, 9, 30, 15, 51, 17, 6, 22)_{\Theta(1,4,9)},$
 $(17, 8, 54, 6, 26, 53, 15, 46, 45, 27, 31, 29, 24)_{\Theta(1,4,9)},$
 $(35, 15, 26, 40, 34, 12, 44, 2, 5, 21, 54, 23, 24)_{\Theta(1,4,9)},$
 $(11, 2, 8, 3, 13, 12, 36, 21, \infty, 19, 48, 0, 29)_{\Theta(1,4,9)},$
 $(\infty, 10, 9, 7, 40, 11, 27, 45, 4, 14, 23, 6, 44)_{\Theta(1,5,8)},$
 $(48, 44, 54, 6, 8, 13, 23, 30, 43, 53, 41, 2, 19)_{\Theta(1,5,8)},$

$(37, 38, 51, 29, 9, 17, 16, 19, 50, 42, 48, 11, 35)_{\Theta(1,5,8)},$
 $(10, 12, 52, 8, 21, 9, 14, 32, 25, 53, 38, 7, 3)_{\Theta(1,5,8)},$
 $(49, 31, 54, 14, 53, 21, 36, 43, 44, 46, 26, 23, 25)_{\Theta(1,5,8)},$
 $(33, 45, 0, 29, 20, 10, 53, 26, 30, 13, 32, 31, 39)_{\Theta(1,5,8)},$
 $(0, 50, 32, 47, 17, 39, 40, 2, 34, 22, 29, 30, 14)_{\Theta(1,5,8)},$
 $(28, 9, 39, 52, 42, 37, 6, 21, 2, 30, 53, 27, 36)_{\Theta(1,5,8)},$
 $(13, 50, 27, 15, 6, 11, 5, 30, 51, 7, 1, 32, 36)_{\Theta(1,5,8)},$
 $(12, 15, 28, 14, 43, 51, 32, 6, 29, 8, \infty, 1, 26)_{\Theta(1,5,8)},$
 $(\infty, 48, 2, 40, 33, 10, 42, 54, 21, 16, 47, 7, 23)_{\Theta(1,6,7)},$
 $(7, 12, 16, 1, 40, 28, 31, 11, 13, 6, 4, 49, 0)_{\Theta(1,6,7)},$
 $(51, 10, 34, 41, 15, 23, 11, 38, 25, 12, 26, 45, 1)_{\Theta(1,6,7)},$
 $(28, 18, 6, 9, 20, 26, 47, 11, 36, 1, 35, 10, 44)_{\Theta(1,6,7)},$
 $(26, 14, 3, 47, 40, 20, 34, 36, 9, 33, 29, 6, 53)_{\Theta(1,6,7)},$
 $(42, 31, 23, 41, 4, 46, 47, 24, 32, 26, 34, 13, 35)_{\Theta(1,6,7)},$
 $(22, 40, 25, 47, 0, 31, 3, 34, 33, 42, 28, 43, 23)_{\Theta(1,6,7)},$
 $(22, 42, 39, 0, 19, 32, 9, 29, 5, 15, 14, 44, 17)_{\Theta(1,6,7)},$
 $(2, 4, 3, 5, 14, 33, 19, 47, 20, 15, 44, 38, 7)_{\Theta(1,6,7)},$
 $(24, 20, 13, 8, 12, 33, 5, 19, 28, 25, 27, 1, \infty)_{\Theta(1,6,7)},$
 $(\infty, 42, 15, 51, 45, 12, 26, 24, 32, 53, 8, 23, 9)_{\Theta(2,3,9)},$
 $(21, 43, 44, 13, 36, 9, 19, 17, 3, 5, 31, 51, 6)_{\Theta(2,3,9)},$
 $(24, 4, 19, 8, 40, 0, 17, 29, 30, 9, 28, 2, 21)_{\Theta(2,3,9)},$
 $(40, 15, 25, 34, 23, 42, 43, 50, 30, 31, 33, 37, 48)_{\Theta(2,3,9)},$
 $(41, 11, 45, 14, 6, 47, 24, 49, 28, 22, 15, 4, 32)_{\Theta(2,3,9)},$
 $(47, 35, 22, 34, 5, 36, 54, 23, 20, 25, 11, 24, 44)_{\Theta(2,3,9)},$
 $(49, 28, 12, 43, 0, 3, 45, 6, 22, 17, 27, 7, 53)_{\Theta(2,3,9)},$
 $(40, 7, 16, 3, 31, 32, 0, 39, 43, 12, 52, 14, 11)_{\Theta(2,3,9)},$
 $(17, 27, 5, 35, 26, 53, 15, 19, 48, 31, 16, 49, 1)_{\Theta(2,3,9)},$
 $(13, 5, 16, 8, 41, 1, 31, 18, 38, \infty, 9, 12, 19)_{\Theta(2,3,9)},$
 $(\infty, 39, 27, 44, 13, 0, 46, 38, 50, 26, 16, 7, 6)_{\Theta(2,5,7)},$
 $(28, 41, 11, 51, 37, 45, 43, 50, 16, 20, 9, 30, 40)_{\Theta(2,5,7)},$
 $(48, 1, 27, 12, 6, 49, 41, 20, 44, 39, 35, 34, 43)_{\Theta(2,5,7)},$
 $(9, 4, 3, 27, 16, 45, 29, 24, 8, 33, 13, 12, 37)_{\Theta(2,5,7)},$
 $(33, 11, 47, 28, 54, 34, 32, 18, 10, 30, 17, 15, 9)_{\Theta(2,5,7)},$
 $(48, 44, 17, 25, 22, 42, 2, 21, 24, 27, 19, 38, 34)_{\Theta(2,5,7)},$
 $(24, 16, 5, 50, 45, 4, 21, 38, 41, 37, 14, 52, 23)_{\Theta(2,5,7)},$
 $(50, 13, 7, 22, 6, 29, 1, 11, 30, 36, 3, 24, 17)_{\Theta(2,5,7)},$
 $(18, 12, 36, 35, 5, 22, 45, 25, 40, 47, 2, 48, 30)_{\Theta(2,5,7)},$
 $(28, 1, 36, 12, 7, 18, 15, 38, 49, 31, 44, 35, \infty)_{\Theta(2,5,7)},$
 $(\infty, 32, 52, 18, 34, 49, 43, 15, 19, 6, 5, 45, 37)_{\Theta(3,3,8)},$
 $(30, 42, 1, 31, 6, 51, 36, 25, 35, 15, 13, 20, 11)_{\Theta(3,3,8)},$
 $(45, 50, 20, 33, 31, 19, 29, 6, 46, 26, 42, 53, 9)_{\Theta(3,3,8)},$
 $(23, 11, 16, 37, 53, 13, 18, 17, 31, 15, 9, 36, 44)_{\Theta(3,3,8)},$
 $(32, 20, 31, 29, 3, 38, 35, 12, 39, 8, 41, 36, 54)_{\Theta(3,3,8)},$
 $(1, 34, 7, 4, 49, 42, 43, 9, 6, 2, 48, 12, 14)_{\Theta(3,3,8)},$
 $(8, 15, 4, 18, 51, 48, 53, 47, 27, 2, 20, 7, 38)_{\Theta(3,3,8)},$
 $(14, 4, 33, 37, 15, 27, 13, 25, 44, 54, 16, 20, 22)_{\Theta(3,3,8)},$
 $(41, 49, 23, 7, 20, 37, 22, 15, 51, 28, 54, 4, 43)_{\Theta(3,3,8)},$

$(44, 18, 15, 10, 53, 26, 0, 22, 7, 17, 45, \infty, 1)_{\Theta(3,3,8)},$
 $(\infty, 48, 43, 54, 32, 0, 8, 14, 31, 20, 23, 42, 50)_{\Theta(3,4,7)},$
 $(8, 22, 7, 35, 45, 20, 2, 12, 6, 10, 36, 3, 48)_{\Theta(3,4,7)},$
 $(5, 27, 39, 43, 29, 1, 14, 7, 34, 30, 52, 41, 17)_{\Theta(3,4,7)},$
 $(53, 16, 34, 31, 30, 13, 25, 19, 41, 36, 29, 7, 10)_{\Theta(3,4,7)},$
 $(47, 33, 24, 19, 35, 29, 40, 9, 39, 21, 3, 28, 0)_{\Theta(3,4,7)},$
 $(18, 7, 21, 37, 5, 10, 26, 31, 33, 38, 11, 19, 4)_{\Theta(3,4,7)},$
 $(19, 14, 21, 53, 12, 24, 34, 45, 30, 31, 10, 9, 0)_{\Theta(3,4,7)},$
 $(48, 23, 41, 3, 49, 6, 47, 27, 20, 10, 30, 14, 32)_{\Theta(3,4,7)},$
 $(34, 40, 43, 2, 3, 11, 21, 8, 52, 44, 42, 37, 16)_{\Theta(3,4,7)},$
 $(2, 0, 1, \infty, 8, 51, 19, 11, 31, 27, 42, 16, 41)_{\Theta(3,4,7)},$
 $(\infty, 19, 54, 21, 22, 25, 53, 14, 13, 36, 18, 17, 26)_{\Theta(3,5,6)},$
 $(37, 7, 17, 30, 8, 14, 41, 49, 43, 45, 46, 51, 26)_{\Theta(3,5,6)},$
 $(1, 2, 10, 49, 25, 33, 24, 27, 12, 48, 7, 39, 13)_{\Theta(3,5,6)},$
 $(21, 54, 44, 10, 11, 24, 42, 33, 0, 9, 50, 7, 14)_{\Theta(3,5,6)},$
 $(19, 14, 1, 4, 49, 48, 31, 2, 25, 45, 16, 38, 3)_{\Theta(3,5,6)},$
 $(37, 9, 33, 45, 21, 27, 5, 7, 10, 25, 32, 47, 13)_{\Theta(3,5,6)},$
 $(2, 54, 16, 13, 40, 1, 45, 27, 52, 19, 31, 25, 50)_{\Theta(3,5,6)},$
 $(43, 18, 48, 25, 10, 0, 47, 46, 40, 45, 14, 49, 11)_{\Theta(3,5,6)},$
 $(13, 21, 53, 6, 50, 49, 20, 1, 37, 41, 45, 3, 33)_{\Theta(3,5,6)},$
 $(4, 0, 28, 38, 23, 7, 17, 41, 42, 21, 8, 6, \infty)_{\Theta(3,5,6)},$
 $(\infty, 7, 51, 17, 34, 54, 2, 8, 9, 5, 25, 44, 53)_{\Theta(4,5,5)},$
 $(38, 8, 25, 0, 36, 18, 14, 48, 3, 17, 5, 21, 51)_{\Theta(4,5,5)},$
 $(28, 3, 21, 30, 32, 9, 36, 41, 43, 12, 27, 40, 2)_{\Theta(4,5,5)},$
 $(25, 39, 11, 24, 2, 29, 18, 20, 51, 54, 1, 4, 19)_{\Theta(4,5,5)},$
 $(50, 38, 17, 48, 9, 28, 5, 11, 21, 6, 39, 1, 30)_{\Theta(4,5,5)},$
 $(27, 2, 34, 4, 53, 38, 1, 16, 50, 14, 24, 35, 30)_{\Theta(4,5,5)},$
 $(6, 29, 14, 22, 11, 52, 20, 2, 43, 38, 51, 18, 15)_{\Theta(4,5,5)},$
 $(44, 3, 35, 29, 30, 20, 16, 51, 22, 32, 7, 42, 11)_{\Theta(4,5,5)},$
 $(45, 22, 29, 34, 41, 33, 8, 11, 17, 30, 23, 40, 32)_{\Theta(4,5,5)},$
 $(37, 3, 51, 50, 40, 14, 46, 42, \infty, 36, 52, 0, 34)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 5 \pmod{55}$, $\infty \mapsto \infty$.

K_{57} Let the vertex set be Z_{57} . The decompositions consist of

$(0, 1, 3, 4, 9, 2, 8, 16, 5, 14, 24, 6, 18)_{\Theta(1,2,11)},$
 $(0, 13, 27, 15, 31, 2, 21, 1, 23, 49, 24, 3, 36)_{\Theta(1,2,11)},$
 $(0, 1, 2, 5, 6, 11, 3, 10, 19, 4, 14, 25, 37)_{\Theta(1,3,10)},$
 $(0, 13, 14, 30, 18, 37, 2, 22, 45, 12, 41, 9, 39)_{\Theta(1,3,10)},$
 $(0, 1, 2, 5, 9, 6, 11, 4, 13, 3, 14, 26, 39)_{\Theta(1,4,9)},$
 $(0, 14, 15, 31, 48, 18, 38, 2, 27, 1, 30, 3, 36)_{\Theta(1,4,9)},$
 $(0, 1, 2, 5, 9, 14, 6, 13, 3, 11, 20, 4, 15)_{\Theta(1,5,8)},$
 $(0, 12, 15, 32, 1, 30, 19, 41, 11, 43, 20, 53, 33)_{\Theta(1,5,8)},$
 $(0, 1, 2, 5, 9, 3, 8, 10, 18, 4, 13, 24, 36)_{\Theta(1,6,7)},$
 $(0, 13, 15, 31, 1, 18, 36, 19, 39, 3, 34, 5, 37)_{\Theta(1,6,7)},$
 $(0, 1, 2, 3, 7, 5, 12, 4, 13, 23, 6, 17, 29)_{\Theta(2,3,9)},$
 $(0, 1, 14, 15, 31, 18, 37, 2, 22, 45, 20, 46, 25)_{\Theta(2,3,9)},$

$(0, 1, 2, 3, 7, 12, 18, 8, 15, 4, 13, 23, 35)_{\Theta(2,5,7)},$
 $(0, 1, 14, 15, 31, 2, 20, 21, 41, 6, 30, 5, 32)_{\Theta(2,5,7)},$
 $(0, 1, 2, 5, 6, 11, 7, 8, 16, 3, 12, 23, 35)_{\Theta(3,3,8)},$
 $(0, 1, 14, 29, 16, 33, 18, 38, 5, 32, 6, 41, 20)_{\Theta(3,3,8)},$
 $(0, 1, 2, 5, 6, 7, 12, 8, 15, 3, 13, 4, 17)_{\Theta(3,4,7)},$
 $(0, 1, 14, 29, 17, 38, 19, 20, 42, 8, 32, 2, 27)_{\Theta(3,4,7)},$
 $(0, 1, 2, 5, 6, 7, 12, 19, 8, 17, 3, 13, 24)_{\Theta(3,5,6)},$
 $(0, 1, 12, 25, 15, 31, 2, 22, 19, 44, 13, 40, 18)_{\Theta(3,5,6)},$
 $(0, 1, 2, 3, 6, 4, 10, 17, 25, 9, 19, 5, 16)_{\Theta(4,5,5)},$
 $(0, 1, 12, 25, 41, 18, 37, 2, 29, 20, 43, 7, 32)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 1 \pmod{57}$.

K_{64} Let the vertex set be $Z_{63} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 21, 23, 59, 31, 15, 27, 1, 48, 52, 16, 26, 61)_{\Theta(1,2,11)},$
 $(4, 30, 0, 23, 58, 29, 44, 34, 31, 49, 2, 9, 32)_{\Theta(1,2,11)},$
 $(48, 2, 10, 19, 62, 20, 49, 54, 11, 12, 37, 51, 8)_{\Theta(1,2,11)},$
 $(44, 23, 10, 59, 16, 5, 2, 32, 49, 11, 45, 53, 47)_{\Theta(1,2,11)},$
 $(53, 35, 55, 61, 0, 12, 49, 33, 51, 13, 4, 56, 25)_{\Theta(1,2,11)},$
 $(46, 61, 11, 37, 6, 59, 17, 0, 55, 48, 4, 50, 31)_{\Theta(1,2,11)},$
 $(28, 27, 57, 2, 33, 37, 32, 35, 54, 13, 26, 17, 21)_{\Theta(1,2,11)},$
 $(46, 58, 13, 50, 8, 18, 32, 48, 47, 62, 31, 56, 0)_{\Theta(1,2,11)},$
 $(6, 8, 12, 9, 22, 60, 45, 52, 38, 43, 15, 42, 28)_{\Theta(1,2,11)},$
 $(46, 20, 43, 7, 8, 57, 58, 12, 50, 38, 40, 45, 56)_{\Theta(1,2,11)},$
 $(47, 40, 24, 33, 4, 27, 38, 8, 39, 45, 21, 9, 3)_{\Theta(1,2,11)},$
 $(46, 10, 1, 41, 56, 24, 21, 2, 13, 8, 49, 14, 27)_{\Theta(1,2,11)},$
 $(37, 59, 41, 14, 29, 22, 12, 42, 0, 50, 33, 54, 36)_{\Theta(1,2,11)},$
 $(2, 57, 18, 41, 13, 62, 26, 29, 20, 32, 53, 31, 30)_{\Theta(1,2,11)},$
 $(55, 36, 47, 31, 57, 0, 54, 3, 1, 33, 52, 53, \infty)_{\Theta(1,2,11)},$
 $(3, 46, 53, 56, 1, 23, 14, 55, \infty, 19, 43, 32, 5)_{\Theta(1,2,11)},$
 $(\infty, 11, 61, 57, 48, 38, 35, 31, 0, 2, 16, 60, 46)_{\Theta(1,3,10)},$
 $(2, 34, 20, 41, 11, 10, 40, 27, 37, 22, 44, 33, 49)_{\Theta(1,3,10)},$
 $(55, 39, 47, 7, 61, 5, 29, 34, 19, 9, 51, 25, 45)_{\Theta(1,3,10)},$
 $(9, 24, 49, 47, 15, 8, 19, 37, 21, 29, 18, 10, 46)_{\Theta(1,3,10)},$
 $(40, 9, 20, 34, 22, 38, 15, 18, 62, 10, 31, 43, 39)_{\Theta(1,3,10)},$
 $(50, 18, 48, 20, 33, 39, 23, 58, 29, 10, 35, 13, 36)_{\Theta(1,3,10)},$
 $(36, 7, 55, 0, 14, 56, 28, 42, 24, 39, 46, 26, 4)_{\Theta(1,3,10)},$
 $(17, 61, 41, 42, 13, 38, 24, 58, 45, 47, 10, 19, 33)_{\Theta(1,3,10)},$
 $(10, 26, 16, 18, 13, 45, 8, 20, 9, 21, 41, 40, 25)_{\Theta(1,3,10)},$
 $(28, 23, 43, 15, 61, 40, 52, 57, 8, 10, 0, 39, 26)_{\Theta(1,3,10)},$
 $(31, 60, 13, 18, 22, 35, 40, 50, 12, 46, 23, 49, 20)_{\Theta(1,3,10)},$
 $(57, 32, 37, 41, 54, 18, 55, 33, 16, 15, 42, 6, 22)_{\Theta(1,3,10)},$
 $(51, 12, 34, 37, 42, 4, 49, 29, 62, 32, 45, 9, 48)_{\Theta(1,3,10)},$
 $(3, 10, 49, 38, 58, 2, 60, 48, 42, 46, 22, 43, 30)_{\Theta(1,3,10)},$
 $(33, 62, 42, 36, 1, 0, 53, 2, 3, 20, 29, \infty, 49)_{\Theta(1,3,10)},$
 $(0, 24, 30, 57, 11, 28, 9, 5, 42, 54, 59, 37, \infty)_{\Theta(1,3,10)},$
 $(\infty, 10, 62, 15, 1, 46, 22, 4, 3, 27, 50, 16, 37)_{\Theta(1,4,9)},$

$(17, 36, 60, 21, 39, 48, 26, 23, 52, 3, 13, 9, 59)_{\Theta(1,4,9)},$
 $(51, 8, 0, 27, 46, 56, 42, 61, 45, 4, 23, 3, 29)_{\Theta(1,4,9)},$
 $(51, 29, 45, 3, 62, 47, 52, 54, 60, 55, 1, 26, 12)_{\Theta(1,4,9)},$
 $(40, 28, 39, 9, 54, 55, 48, 22, 61, 32, 34, 26, 15)_{\Theta(1,4,9)},$
 $(8, 49, 27, 11, 22, 40, 6, 57, 9, 48, 34, 2, 60)_{\Theta(1,4,9)},$
 $(11, 46, 51, 7, 49, 4, 14, 8, 6, 32, 15, 10, 33)_{\Theta(1,4,9)},$
 $(11, 19, 3, 55, 30, 47, 48, 61, 7, 31, 0, 46, 13)_{\Theta(1,4,9)},$
 $(49, 24, 31, 42, 16, 52, 17, 47, 30, 44, 57, 45, 23)_{\Theta(1,4,9)},$
 $(15, 9, 16, 32, 7, 22, 26, 19, 60, 48, 6, 52, 27)_{\Theta(1,4,9)},$
 $(18, 61, 50, 22, 14, 24, 39, 35, 0, 16, 40, 9, 19)_{\Theta(1,4,9)},$
 $(6, 24, 14, 29, 28, 23, 61, 41, 21, 19, 55, 42, 11)_{\Theta(1,4,9)},$
 $(7, 37, 36, 44, 34, 41, 18, 8, 26, 56, 62, 21, 30)_{\Theta(1,4,9)},$
 $(12, 38, 31, 4, 8, 3, 10, 58, 46, 44, 16, 56, 36)_{\Theta(1,4,9)},$
 $(56, 61, 33, 5, 8, 10, 39, 53, 44, 55, 52, 36, \infty)_{\Theta(1,4,9)},$
 $(2, 39, \infty, 0, 48, 29, 34, 6, 7, 14, 24, 12, 60)_{\Theta(1,4,9)},$
 $(\infty, 9, 49, 16, 5, 35, 47, 46, 57, 20, 10, 19, 23)_{\Theta(1,5,8)},$
 $(2, 11, 36, 0, 17, 18, 50, 49, 55, 9, 40, 10, 26)_{\Theta(1,5,8)},$
 $(41, 29, 53, 10, 54, 16, 42, 14, 22, 0, 3, 9, 39)_{\Theta(1,5,8)},$
 $(33, 53, 30, 4, 39, 34, 10, 22, 5, 41, 14, 18, 59)_{\Theta(1,5,8)},$
 $(21, 14, 7, 23, 46, 1, 45, 20, 6, 53, 47, 34, 19)_{\Theta(1,5,8)},$
 $(36, 44, 40, 26, 57, 1, 41, 49, 27, 6, 62, 2, 9)_{\Theta(1,5,8)},$
 $(6, 58, 56, 40, 29, 48, 51, 32, 21, 23, 24, 14, 12)_{\Theta(1,5,8)},$
 $(62, 45, 30, 12, 10, 38, 54, 18, 47, 6, 26, 49, 11)_{\Theta(1,5,8)},$
 $(11, 42, 35, 55, 24, 2, 43, 45, 7, 52, 9, 17, 38)_{\Theta(1,5,8)},$
 $(7, 59, 1, 62, 27, 4, 60, 57, 34, 33, 40, 49, 29)_{\Theta(1,5,8)},$
 $(40, 18, 34, 9, 4, 43, 22, 6, 30, 51, 55, 46, 20)_{\Theta(1,5,8)},$
 $(50, 36, 35, 1, 60, 9, 29, 19, 58, 0, 44, 59, 10)_{\Theta(1,5,8)},$
 $(53, 51, 3, 61, 24, 29, 39, 60, 44, 50, 59, 36, 17)_{\Theta(1,5,8)},$
 $(6, 24, 35, 54, 33, 8, 10, 25, 7, 53, 36, 27, 60)_{\Theta(1,5,8)},$
 $(62, 29, 59, 47, 60, 5, 33, 61, 51, 0, 42, 10, \infty)_{\Theta(1,5,8)},$
 $(0, 12, 9, 45, 34, 49, 60, \infty, 55, 31, 44, 43, 15)_{\Theta(1,5,8)},$
 $(\infty, 51, 43, 62, 52, 33, 30, 42, 16, 53, 4, 5, 60)_{\Theta(1,6,7)},$
 $(33, 36, 60, 2, 34, 32, 44, 58, 42, 26, 27, 41, 37)_{\Theta(1,6,7)},$
 $(46, 20, 8, 18, 50, 26, 2, 56, 60, 14, 40, 58, 35)_{\Theta(1,6,7)},$
 $(46, 25, 40, 62, 7, 41, 13, 18, 33, 10, 43, 39, 17)_{\Theta(1,6,7)},$
 $(38, 35, 62, 53, 46, 28, 8, 55, 24, 25, 47, 7, 3)_{\Theta(1,6,7)},$
 $(23, 33, 59, 53, 21, 12, 20, 45, 41, 28, 49, 25, 54)_{\Theta(1,6,7)},$
 $(13, 55, 18, 15, 26, 46, 44, 22, 57, 42, 29, 11, 14)_{\Theta(1,6,7)},$
 $(22, 17, 45, 20, 26, 9, 5, 0, 58, 2, 36, 48, 42)_{\Theta(1,6,7)},$
 $(60, 16, 30, 17, 36, 57, 45, 49, 47, 29, 33, 8, 32)_{\Theta(1,6,7)},$
 $(32, 17, 45, 31, 52, 34, 1, 9, 58, 47, 16, 41, 12)_{\Theta(1,6,7)},$
 $(43, 42, 23, 21, 28, 57, 9, 6, 25, 12, 7, 56, 50)_{\Theta(1,6,7)},$
 $(47, 15, 1, 6, 33, 26, 17, 12, 45, 2, 29, 19, 62)_{\Theta(1,6,7)},$
 $(3, 57, 10, 16, 31, 49, 11, 32, 48, 25, 55, 2, 43)_{\Theta(1,6,7)},$
 $(40, 24, 3, 46, 57, 31, 7, 54, 42, 30, 27, 0, 35)_{\Theta(1,6,7)},$
 $(0, 10, 44, 57, 50, 48, 45, 9, 55, 62, 47, \infty, 46)_{\Theta(1,6,7)},$
 $(3, 5, 2, 10, \infty, 6, 49, 55, 16, 51, 57, 34, 35)_{\Theta(1,6,7)},$

$(\infty, 45, 22, 44, 10, 19, 25, 49, 46, 60, 52, 9, 11)_{\Theta(2,3,9)},$
 $(58, 44, 53, 6, 40, 34, 49, 41, 43, 16, 21, 7, 36)_{\Theta(2,3,9)},$
 $(52, 14, 3, 33, 51, 32, 13, 55, 9, 44, 62, 36, 42)_{\Theta(2,3,9)},$
 $(34, 7, 10, 11, 50, 38, 39, 51, 45, 57, 24, 6, 52)_{\Theta(2,3,9)},$
 $(56, 60, 37, 4, 33, 24, 14, 55, 36, 27, 59, 46, 15)_{\Theta(2,3,9)},$
 $(50, 53, 46, 30, 27, 28, 52, 11, 21, 47, 35, 23, 16)_{\Theta(2,3,9)},$
 $(3, 39, 41, 12, 11, 19, 9, 42, 2, 52, 46, 16, 54)_{\Theta(2,3,9)},$
 $(22, 23, 40, 16, 38, 36, 57, 62, 15, 7, 5, 12, 13)_{\Theta(2,3,9)},$
 $(58, 24, 27, 59, 47, 16, 7, 3, 13, 61, 30, 8, 17)_{\Theta(2,3,9)},$
 $(10, 45, 12, 35, 18, 62, 5, 27, 11, 1, 61, 47, 60)_{\Theta(2,3,9)},$
 $(41, 26, 13, 28, 15, 53, 36, 21, 25, 46, 8, 11, 6)_{\Theta(2,3,9)},$
 $(26, 23, 37, 5, 36, 4, 13, 51, 50, 62, 0, 47, 39)_{\Theta(2,3,9)},$
 $(27, 6, 20, 7, 39, 50, 52, 8, 3, 40, 21, 57, 33)_{\Theta(2,3,9)},$
 $(1, 47, 57, 29, 44, 30, 28, 49, 11, 54, 26, 31, 14)_{\Theta(2,3,9)},$
 $(31, 14, 15, 58, 19, 23, 7, 41, 37, 18, 0, 6, \infty)_{\Theta(2,3,9)},$
 $(0, 20, 56, 54, 50, 40, 15, 4, \infty, 24, 3, 29, 12)_{\Theta(2,3,9)},$
 $(\infty, 52, 57, 32, 10, 31, 38, 19, 14, 25, 8, 7, 24)_{\Theta(2,5,7)},$
 $(37, 5, 62, 56, 23, 41, 22, 13, 60, 58, 48, 0, 57)_{\Theta(2,5,7)},$
 $(28, 12, 50, 11, 25, 18, 56, 7, 43, 30, 35, 24, 21)_{\Theta(2,5,7)},$
 $(12, 46, 43, 47, 39, 50, 16, 15, 35, 11, 36, 17, 13)_{\Theta(2,5,7)},$
 $(12, 30, 2, 41, 32, 36, 22, 27, 45, 33, 13, 24, 28)_{\Theta(2,5,7)},$
 $(32, 23, 1, 24, 50, 2, 40, 47, 5, 56, 11, 13, 39)_{\Theta(2,5,7)},$
 $(18, 33, 38, 21, 29, 6, 9, 19, 10, 34, 2, 11, 53)_{\Theta(2,5,7)},$
 $(61, 47, 34, 35, 50, 59, 57, 22, 45, 48, 1, 6, 25)_{\Theta(2,5,7)},$
 $(5, 19, 16, 38, 25, 53, 59, 52, 28, 35, 3, 37, 49)_{\Theta(2,5,7)},$
 $(24, 11, 51, 25, 35, 22, 40, 49, 15, 57, 18, 41, 30)_{\Theta(2,5,7)},$
 $(58, 54, 52, 32, 17, 53, 47, 40, 56, 9, 55, 51, 50)_{\Theta(2,5,7)},$
 $(0, 43, 23, 34, 8, 38, 55, 41, 62, 42, 7, 52, 37)_{\Theta(2,5,7)},$
 $(4, 38, 22, 55, 54, 5, 13, 35, 49, 26, 52, 62, 6)_{\Theta(2,5,7)},$
 $(40, 52, 44, 62, 4, 38, 51, 53, 43, 15, 48, 35, 9)_{\Theta(2,5,7)},$
 $(28, 13, 27, 37, 25, 21, 48, 34, 36, 43, 16, 2, \infty)_{\Theta(2,5,7)},$
 $(2, 3, 44, 60, 33, 35, \infty, 58, 26, 45, 57, 48, 56)_{\Theta(2,5,7)},$
 $(\infty, 15, 57, 7, 3, 19, 41, 40, 8, 9, 23, 33, 13)_{\Theta(3,3,8)},$
 $(10, 57, 37, 4, 23, 44, 39, 6, 62, 2, 5, 41, 27)_{\Theta(3,3,8)},$
 $(33, 42, 40, 18, 28, 37, 4, 48, 0, 25, 30, 49, 24)_{\Theta(3,3,8)},$
 $(55, 28, 46, 32, 35, 42, 30, 18, 21, 31, 13, 11, 57)_{\Theta(3,3,8)},$
 $(0, 46, 53, 47, 15, 43, 55, 49, 12, 24, 58, 11, 22)_{\Theta(3,3,8)},$
 $(1, 34, 16, 6, 15, 58, 32, 40, 61, 38, 55, 14, 60)_{\Theta(3,3,8)},$
 $(25, 41, 4, 54, 16, 33, 53, 26, 1, 57, 40, 6, 15)_{\Theta(3,3,8)},$
 $(59, 36, 3, 58, 35, 38, 45, 23, 15, 55, 50, 14, 10)_{\Theta(3,3,8)},$
 $(57, 42, 53, 59, 17, 16, 35, 5, 7, 23, 55, 4, 60)_{\Theta(3,3,8)},$
 $(42, 44, 36, 12, 13, 9, 22, 40, 62, 24, 4, 45, 15)_{\Theta(3,3,8)},$
 $(57, 2, 3, 35, 51, 45, 6, 58, 56, 40, 21, 42, 14)_{\Theta(3,3,8)},$
 $(27, 36, 6, 17, 43, 33, 32, 59, 4, 44, 40, 2, 9)_{\Theta(3,3,8)},$
 $(19, 26, 58, 41, 52, 61, 8, 50, 3, 45, 30, 4, 20)_{\Theta(3,3,8)},$
 $(9, 41, 11, 42, 49, 38, 53, 5, 56, 6, 51, 33, 31)_{\Theta(3,3,8)},$
 $(39, 24, 19, 5, 57, 52, 28, 55, 31, 26, 37, \infty, 11)_{\Theta(3,3,8)},$

$(3, 31, 4, 27, 18, 43, 34, 15, 14, 54, 0, \infty, 5)_{\Theta(3,3,8)},$
 $(\infty, 62, 37, 40, 22, 56, 42, 21, 46, 14, 45, 10, 36)_{\Theta(3,4,7)},$
 $(53, 28, 58, 12, 37, 19, 7, 54, 18, 41, 62, 38, 55)_{\Theta(3,4,7)},$
 $(9, 36, 21, 28, 17, 39, 60, 29, 50, 40, 13, 42, 25)_{\Theta(3,4,7)},$
 $(50, 5, 17, 46, 57, 13, 11, 18, 33, 19, 42, 47, 29)_{\Theta(3,4,7)},$
 $(20, 9, 5, 8, 39, 31, 62, 46, 18, 28, 50, 15, 11)_{\Theta(3,4,7)},$
 $(12, 59, 62, 34, 52, 61, 41, 56, 32, 19, 53, 36, 44)_{\Theta(3,4,7)},$
 $(22, 1, 16, 59, 60, 51, 30, 20, 8, 12, 5, 31, 10)_{\Theta(3,4,7)},$
 $(58, 39, 44, 28, 8, 59, 45, 25, 32, 51, 24, 22, 35)_{\Theta(3,4,7)},$
 $(61, 13, 6, 10, 28, 25, 22, 35, 8, 14, 44, 7, 51)_{\Theta(3,4,7)},$
 $(29, 60, 45, 42, 13, 47, 59, 54, 23, 16, 5, 36, 46)_{\Theta(3,4,7)},$
 $(10, 17, 25, 51, 26, 31, 56, 0, 61, 2, 13, 60, 6)_{\Theta(3,4,7)},$
 $(23, 16, 55, 54, 58, 22, 7, 1, 41, 31, 18, 13, 20)_{\Theta(3,4,7)},$
 $(19, 13, 17, 28, 54, 11, 62, 38, 42, 24, 60, 27, 14)_{\Theta(3,4,7)},$
 $(14, 19, 37, 13, 5, 60, 9, 57, 40, 61, 50, 55, 58)_{\Theta(3,4,7)},$
 $(28, 45, 34, 1, 23, 8, 22, 0, 17, 24, 2, 20, \infty)_{\Theta(3,4,7)},$
 $(3, 55, 16, 14, 46, 1, 0, 2, 59, 26, \infty, 32, 9)_{\Theta(3,4,7)},$
 $(\infty, 5, 45, 19, 32, 51, 3, 30, 16, 9, 11, 26, 15)_{\Theta(3,5,6)},$
 $(13, 6, 39, 53, 40, 33, 52, 46, 30, 44, 36, 51, 7)_{\Theta(3,5,6)},$
 $(1, 44, 5, 56, 22, 45, 8, 13, 25, 14, 34, 50, 38)_{\Theta(3,5,6)},$
 $(38, 57, 30, 10, 40, 55, 36, 34, 17, 50, 39, 47, 60)_{\Theta(3,5,6)},$
 $(58, 0, 15, 46, 21, 61, 30, 19, 48, 52, 50, 4, 57)_{\Theta(3,5,6)},$
 $(24, 14, 21, 22, 6, 30, 31, 59, 54, 19, 18, 44, 43)_{\Theta(3,5,6)},$
 $(24, 19, 31, 6, 49, 11, 47, 22, 32, 53, 18, 36, 56)_{\Theta(3,5,6)},$
 $(1, 49, 37, 19, 36, 0, 30, 41, 34, 39, 7, 20, 10)_{\Theta(3,5,6)},$
 $(40, 23, 17, 20, 56, 10, 59, 7, 6, 5, 51, 4, 33)_{\Theta(3,5,6)},$
 $(36, 3, 17, 8, 27, 48, 39, 44, 5, 26, 50, 28, 55)_{\Theta(3,5,6)},$
 $(10, 25, 5, 7, 42, 21, 28, 32, 27, 41, 60, 54, 49)_{\Theta(3,5,6)},$
 $(31, 44, 4, 47, 35, 25, 13, 48, 18, 43, 26, 38, 53)_{\Theta(3,5,6)},$
 $(23, 39, 29, 43, 58, 13, 46, 17, 44, 21, 16, 41, 61)_{\Theta(3,5,6)},$
 $(10, 14, 44, 42, 34, 32, 31, 62, 53, 2, 15, 27, 1)_{\Theta(3,5,6)},$
 $(11, 62, 44, 40, 51, 42, 13, 5, 14, 0, 15, 8, \infty)_{\Theta(3,5,6)},$
 $(0, 1, 10, 23, 41, 34, 28, 19, \infty, 47, 38, 54, 30)_{\Theta(3,5,6)},$
 $(\infty, 44, 25, 22, 20, 58, 50, 17, 43, 12, 26, 60, 53)_{\Theta(4,5,5)},$
 $(10, 23, \infty, 48, 19, 56, 3, 35, 30, 41, 34, 28, 1)_{\Theta(4,5,5)},$
 $(58, 19, 33, 23, 0, 43, 62, 11, 21, 39, 14, 7, 41)_{\Theta(4,5,5)},$
 $(9, 45, 38, 54, 1, 11, 36, 28, 39, 52, 49, 34, 20)_{\Theta(4,5,5)},$
 $(13, 5, 17, 53, 24, 46, 49, 43, 3, 14, 58, 15, 29)_{\Theta(4,5,5)},$
 $(30, 38, 28, 44, 26, 1, 10, 32, 22, 24, 42, 20, 5)_{\Theta(4,5,5)},$
 $(25, 26, 24, 0, 54, 4, 47, 1, 22, 51, 52, 32, 13)_{\Theta(4,5,5)},$
 $(44, 29, 5, 4, 22, 6, 32, 9, 62, 23, 41, 18, 46)_{\Theta(4,5,5)},$
 $(11, 29, 50, 62, 60, 22, 7, 54, 49, 44, 33, 27, 17)_{\Theta(4,5,5)},$
 $(42, 20, 18, 35, 57, 12, 49, 58, 23, 17, 3, 53, 37)_{\Theta(4,5,5)},$
 $(19, 36, 45, 10, 23, 12, 4, 40, 13, 20, 48, 35, 49)_{\Theta(4,5,5)},$
 $(21, 59, 48, 39, 51, 41, 50, 49, 7, 61, 19, 4, 62)_{\Theta(4,5,5)},$
 $(11, 22, 33, 41, 57, 5, 36, 7, 19, 60, 2, 16, 27)_{\Theta(4,5,5)},$
 $(50, 2, 45, 60, 28, 5, 43, 54, 34, 52, 6, 48, 24)_{\Theta(4,5,5)},$

$(3, 12, 10, 31, 58, 26, 17, 21, 25, 48, 32, 36, 9)_{\Theta(4,5,5)},$
 $(0, 34, 35, 53, 45, 51, 3, 61, 30, \infty, 15, 9, 42)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 7 \pmod{63}$, $\infty \mapsto \infty$.

K_{77} Let the vertex set be $Z_{76} \cup \{\infty\}$. The decompositions consist of the graphs

$(0, 1, \infty, 68, 54, 56, 10, 7, 42, 40, 71, 27, 73)_{\Theta(1,2,11)},$
 $(32, 61, 0, 45, 6, 25, 49, 38, 11, 36, 48, 31, 40)_{\Theta(1,2,11)},$
 $(9, 35, 36, 1, 41, 42, 3, 25, 23, 47, 13, 75, 61)_{\Theta(1,2,11)},$
 $(7, 2, 61, 67, 40, 1, 70, 23, 0, 69, 36, 6, 56)_{\Theta(1,2,11)},$
 $(63, 20, 15, 70, 61, 4, 38, 41, 9, 68, 75, 29, 62)_{\Theta(1,2,11)},$
 $(12, 26, 62, 23, 36, 18, 28, 25, 9, 37, 17, 59, 50)_{\Theta(1,2,11)},$
 $(51, 4, 13, 43, 18, 65, 2, 23, 19, 37, 47, 41, 52)_{\Theta(1,2,11)},$
 $(3, 67, 34, 1, 24, 2, 29, 14, 37, 49, 18, 69, 11)_{\Theta(1,2,11)},$
 $(51, 12, 57, 61, 26, 18, 71, 16, 21, 42, 43, 7, 22)_{\Theta(1,2,11)},$
 $(5, 40, 56, 10, 38, 58, 70, 66, 50, 44, 63, 46, 2)_{\Theta(1,2,11)},$
 $(67, 52, 32, 54, 43, 40, 0, 4, 28, 22, 3, \infty, 70)_{\Theta(1,2,11)},$
 $(0, 1, \infty, 33, 67, 36, 53, 49, 26, 72, 45, 19, 38)_{\Theta(1,3,10)},$
 $(43, 63, 47, 57, 51, 48, 44, 18, 10, 9, 4, 20, 40)_{\Theta(1,3,10)},$
 $(38, 40, 2, 74, 47, 42, 26, 3, 37, 72, 8, 66, 69)_{\Theta(1,3,10)},$
 $(9, 1, 24, 66, 26, 14, 43, 31, 18, 49, 4, 22, 13)_{\Theta(1,3,10)},$
 $(41, 8, 64, 63, 44, 4, 48, 42, 29, 13, 33, 20, 59)_{\Theta(1,3,10)},$
 $(19, 22, 5, 39, 30, 4, 52, 61, 9, 50, 74, 35, 65)_{\Theta(1,3,10)},$
 $(23, 39, 75, 12, 64, 7, 36, 50, 21, 70, 13, 18, 32)_{\Theta(1,3,10)},$
 $(45, 75, 38, 32, 8, 67, 73, 25, 4, 6, 31, 29, 27)_{\Theta(1,3,10)},$
 $(21, 28, 43, 50, 57, 67, 27, 26, 54, 29, 48, 40, 18)_{\Theta(1,3,10)},$
 $(10, 0, 30, 15, 51, 69, 54, 8, 33, 71, 39, 13, 24)_{\Theta(1,3,10)},$
 $(12, 50, 7, 29, 23, 2, 34, 3, 17, 35, 62, 19, \infty)_{\Theta(1,3,10)},$
 $(0, 1, \infty, 6, 45, 30, 65, 49, 64, 24, 41, 58, 15)_{\Theta(1,4,9)},$
 $(5, 75, 35, 52, 38, 10, 26, 16, 69, 67, 12, 14, 15)_{\Theta(1,4,9)},$
 $(14, 54, 36, 26, 34, 10, 60, 25, 0, 31, 75, 24, 5)_{\Theta(1,4,9)},$
 $(9, 36, 33, 35, 40, 67, 5, 53, 16, 49, 69, 18, 63)_{\Theta(1,4,9)},$
 $(57, 34, 3, 62, 43, 19, 66, 9, 27, 36, 74, 10, 64)_{\Theta(1,4,9)},$
 $(64, 67, 62, 14, 13, 9, 40, 11, 38, 20, 19, 71, 52)_{\Theta(1,4,9)},$
 $(17, 57, 13, 60, 65, 20, 55, 49, 62, 51, 28, 37, 47)_{\Theta(1,4,9)},$
 $(11, 62, 24, 35, 7, 61, 6, 59, 67, 21, 54, 2, 20)_{\Theta(1,4,9)},$
 $(33, 30, 2, 43, 1, 42, 53, 11, 67, 60, 28, 4, 74)_{\Theta(1,4,9)},$
 $(54, 59, 61, 49, 23, 67, 63, 66, 52, 72, 24, 36, 20)_{\Theta(1,4,9)},$
 $(37, 24, 44, 52, 58, 22, 7, 14, 40, 29, \infty, 3, 67)_{\Theta(1,4,9)},$
 $(0, 1, \infty, 6, 5, 26, 47, 67, 27, 14, 54, 60, 35)_{\Theta(1,5,8)},$
 $(9, 57, 69, 50, 70, 51, 42, 64, 72, 21, 11, 34, 65)_{\Theta(1,5,8)},$
 $(16, 22, 74, 7, 46, 8, 25, 49, 72, 53, 39, 14, 37)_{\Theta(1,5,8)},$
 $(49, 37, 15, 72, 0, 64, 5, 27, 44, 42, 16, 53, 51)_{\Theta(1,5,8)},$
 $(68, 20, 14, 1, 36, 70, 52, 5, 63, 9, 39, 66, 59)_{\Theta(1,5,8)},$
 $(26, 10, 54, 43, 12, 55, 29, 49, 23, 27, 20, 53, 14)_{\Theta(1,5,8)},$
 $(12, 36, 47, 15, 5, 54, 48, 33, 16, 9, 74, 30, 27)_{\Theta(1,5,8)},$
 $(47, 20, 14, 60, 30, 31, 41, 28, 72, 58, 49, 19, 17)_{\Theta(1,5,8)},$
 $(41, 3, 36, 31, 59, 67, 37, 42, 47, 24, 27, 40, 61)_{\Theta(1,5,8)},$

$(31, 55, 46, 58, 6, 14, 47, 32, 52, 51, 72, 41, 34)_{\Theta(1,5,8)},$
 $(32, 42, 22, 56, 45, 71, 34, 5, 41, 6, 23, \infty, 25)_{\Theta(1,5,8)},$
 $(0, 1, \infty, 43, 15, 23, 37, 60, 62, 57, 26, 70, 24)_{\Theta(1,6,7)},$
 $(38, 15, 53, 55, 75, 56, 3, 0, 71, 45, 4, 70, 61)_{\Theta(1,6,7)},$
 $(23, 14, 34, 52, 35, 8, 63, 66, 38, 39, 7, 32, 0)_{\Theta(1,6,7)},$
 $(49, 25, 28, 9, 69, 50, 16, 45, 20, 12, 27, 6, 75)_{\Theta(1,6,7)},$
 $(50, 24, 19, 34, 16, 66, 10, 13, 2, 74, 21, 18, 35)_{\Theta(1,6,7)},$
 $(17, 60, 46, 16, 56, 61, 48, 42, 2, 54, 15, 34, 1)_{\Theta(1,6,7)},$
 $(71, 11, 74, 0, 72, 6, 33, 42, 50, 75, 8, 56, 63)_{\Theta(1,6,7)},$
 $(20, 14, 9, 41, 34, 39, 26, 19, 32, 5, 15, 11, 13)_{\Theta(1,6,7)},$
 $(8, 5, 39, 45, 51, 9, 17, 32, 3, 41, 69, 15, 12)_{\Theta(1,6,7)},$
 $(4, 39, 47, 1, 40, 69, 49, 46, 24, 30, 52, 21, 3)_{\Theta(1,6,7)},$
 $(71, 37, 57, 70, 35, 17, 58, 32, 12, 73, 14, 30, \infty)_{\Theta(1,6,7)},$
 $(0, 1, \infty, 65, 42, 74, 72, 50, 71, 58, 44, 67, 27)_{\Theta(2,3,9)},$
 $(6, 25, 5, 20, 60, 53, 0, 58, 39, 17, 22, 63, 67)_{\Theta(2,3,9)},$
 $(65, 66, 23, 69, 0, 28, 55, 72, 64, 68, 49, 31, 7)_{\Theta(2,3,9)},$
 $(39, 56, 11, 60, 63, 62, 68, 10, 55, 20, 8, 42, 2)_{\Theta(2,3,9)},$
 $(13, 20, 63, 62, 53, 49, 21, 65, 64, 32, 2, 31, 70)_{\Theta(2,3,9)},$
 $(59, 53, 50, 72, 34, 8, 37, 47, 49, 62, 18, 11, 10)_{\Theta(2,3,9)},$
 $(47, 46, 61, 9, 54, 41, 29, 7, 72, 12, 37, 45, 40)_{\Theta(2,3,9)},$
 $(19, 54, 51, 24, 33, 5, 30, 10, 38, 43, 70, 44, 65)_{\Theta(2,3,9)},$
 $(64, 3, 63, 30, 18, 49, 4, 23, 34, 38, 62, 32, 71)_{\Theta(2,3,9)},$
 $(52, 3, 32, 49, 33, 25, 55, 67, 42, 58, 65, 41, 59)_{\Theta(2,3,9)},$
 $(68, 17, 7, 20, 11, 44, 54, 37, 24, 41, 2, \infty, 19)_{\Theta(2,3,9)},$
 $(0, 1, \infty, 22, 11, 26, 8, 36, 71, 3, 54, 40, 2)_{\Theta(2,5,7)},$
 $(43, 42, 32, 14, 63, 22, 18, 0, 30, 72, 12, 57, 69)_{\Theta(2,5,7)},$
 $(50, 49, 59, 34, 11, 39, 51, 19, 56, 53, 3, 72, 21)_{\Theta(2,5,7)},$
 $(40, 1, 30, 13, 75, 4, 58, 23, 52, 29, 7, 68, 44)_{\Theta(2,5,7)},$
 $(48, 51, 60, 27, 43, 1, 47, 0, 42, 34, 36, 23, 24)_{\Theta(2,5,7)},$
 $(18, 16, 53, 74, 31, 38, 2, 9, 56, 12, 6, 34, 1)_{\Theta(2,5,7)},$
 $(26, 40, 63, 24, 75, 69, 10, 7, 53, 74, 19, 5, 43)_{\Theta(2,5,7)},$
 $(54, 1, 57, 66, 71, 27, 53, 61, 48, 56, 52, 32, 37)_{\Theta(2,5,7)},$
 $(5, 45, 13, 65, 59, 40, 49, 10, 60, 25, 8, 14, 27)_{\Theta(2,5,7)},$
 $(64, 61, 46, 19, 29, 63, 7, 9, 28, 2, 65, 42, 59)_{\Theta(2,5,7)},$
 $(49, 62, 18, 60, 61, 3, \infty, 10, 7, 47, 23, 22, 73)_{\Theta(2,5,7)},$
 $(0, 1, \infty, 18, 57, 54, 21, 44, 38, 65, 2, 71, 30)_{\Theta(3,3,8)},$
 $(27, 32, 4, 7, 14, 12, 46, 26, 22, 43, 35, 69, 20)_{\Theta(3,3,8)},$
 $(67, 68, 12, 21, 43, 66, 24, 74, 44, 55, 27, 39, 9)_{\Theta(3,3,8)},$
 $(29, 26, 51, 20, 67, 71, 3, 50, 32, 73, 22, 47, 56)_{\Theta(3,3,8)},$
 $(25, 57, 9, 53, 64, 56, 27, 54, 69, 33, 0, 32, 39)_{\Theta(3,3,8)},$
 $(45, 48, 23, 72, 43, 53, 71, 35, 1, 64, 61, 5, 6)_{\Theta(3,3,8)},$
 $(58, 1, 20, 10, 48, 8, 6, 38, 66, 71, 27, 64, 13)_{\Theta(3,3,8)},$
 $(51, 4, 66, 58, 57, 62, 34, 20, 36, 31, 74, 53, 46)_{\Theta(3,3,8)},$
 $(75, 63, 60, 45, 74, 7, 65, 51, 64, 53, 42, 5, 57)_{\Theta(3,3,8)},$
 $(12, 21, 38, 35, 74, 62, 8, 56, 27, 68, 14, 33, 66)_{\Theta(3,3,8)},$
 $(1, 67, 29, 21, 32, 51, \infty, 3, 4, 63, 26, 42, 2)_{\Theta(3,3,8)},$

$(0, 1, \infty, 27, 22, 12, 69, 64, 46, 71, 58, 59, 19)_{\Theta(3,4,7)},$
 $(69, 68, 47, 74, 50, 53, 18, 49, 1, 48, 11, 13, 45)_{\Theta(3,4,7)},$
 $(44, 3, 34, 4, 15, 49, 22, 68, 11, 41, 27, 72, 74)_{\Theta(3,4,7)},$
 $(30, 58, 42, 5, 7, 51, 45, 67, 71, 6, 46, 14, 73)_{\Theta(3,4,7)},$
 $(25, 41, 12, 46, 49, 63, 68, 59, 71, 1, 61, 50, 29)_{\Theta(3,4,7)},$
 $(32, 30, 0, 37, 53, 13, 16, 49, 40, 55, 73, 68, 51)_{\Theta(3,4,7)},$
 $(52, 26, 50, 59, 37, 68, 10, 31, 57, 6, 35, 0, 43)_{\Theta(3,4,7)},$
 $(64, 62, 11, 14, 44, 52, 53, 29, 67, 1, 23, 13, 58)_{\Theta(3,4,7)},$
 $(7, 40, 35, 15, 14, 55, 31, 22, 2, 36, 8, 62, 10)_{\Theta(3,4,7)},$
 $(6, 31, 12, 39, 49, 2, 47, 14, 52, 1, 8, 70, 20)_{\Theta(3,4,7)},$
 $(11, 28, 24, 31, 57, 68, 32, 9, 5, 6, \infty, 1, 44)_{\Theta(3,4,7)},$
 $(0, 1, \infty, 35, 75, 46, 20, 51, 48, 72, 37, 28, 30)_{\Theta(3,5,6)},$
 $(33, 21, 3, 32, 54, 16, 75, 23, 42, 41, 14, 2, 34)_{\Theta(3,5,6)},$
 $(63, 68, 22, 37, 19, 65, 59, 3, 10, 40, 21, 17, 62)_{\Theta(3,5,6)},$
 $(34, 33, 4, 25, 27, 31, 26, 11, 71, 16, 30, 72, 28)_{\Theta(3,5,6)},$
 $(70, 24, 5, 44, 19, 56, 64, 60, 27, 15, 14, 38, 47)_{\Theta(3,5,6)},$
 $(74, 8, 66, 62, 64, 30, 19, 68, 52, 67, 9, 56, 5)_{\Theta(3,5,6)},$
 $(16, 54, 4, 70, 22, 39, 49, 29, 23, 56, 38, 73, 21)_{\Theta(3,5,6)},$
 $(50, 13, 70, 30, 64, 7, 10, 49, 23, 51, 41, 59, 61)_{\Theta(3,5,6)},$
 $(69, 37, 5, 43, 7, 38, 19, 44, 25, 41, 27, 11, 3)_{\Theta(3,5,6)},$
 $(33, 42, 55, 68, 56, 73, 23, 63, 38, 45, 44, 62, 65)_{\Theta(3,5,6)},$
 $(11, 4, 20, 53, 74, 26, \infty, 17, 8, 6, 25, 40, 75)_{\Theta(3,5,6)},$
 $(0, 1, \infty, 15, 7, 2, 73, 26, 27, 55, 3, 75, 19)_{\Theta(4,5,5)},$
 $(4, 8, 71, 36, 24, 13, 66, 50, 57, 34, 74, 16, 44)_{\Theta(4,5,5)},$
 $(59, 54, 69, 12, 6, 15, 68, 39, 65, 0, 53, 38, 67)_{\Theta(4,5,5)},$
 $(71, 73, 44, 12, 36, 40, 48, 15, 57, 46, 33, 35, 74)_{\Theta(4,5,5)},$
 $(45, 52, 13, 46, 9, 62, 53, 67, 58, 10, 27, 17, 32)_{\Theta(4,5,5)},$
 $(25, 50, 12, 33, 71, 17, 28, 69, 29, 55, 56, 52, 42)_{\Theta(4,5,5)},$
 $(18, 26, 11, 54, 40, 62, 52, 47, 50, 75, 64, 3, 0)_{\Theta(4,5,5)},$
 $(70, 56, 14, 36, 54, 20, 65, 48, 49, 32, 51, 10, 61)_{\Theta(4,5,5)},$
 $(61, 54, 30, 35, 23, 64, 10, 63, 12, 41, 38, 50, 39)_{\Theta(4,5,5)},$
 $(29, 21, 25, 6, 33, 15, 31, 9, 67, 71, 1, 48, 73)_{\Theta(4,5,5)},$
 $(21, 36, 49, 27, 75, 19, 55, 48, 6, \infty, 46, 50, 23)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 4 \pmod{76}$, $\infty \mapsto \infty$.

K_{92} Let the vertex set be Z_{92} . The decompositions consist of the graphs

$(63, 36, 7, 78, 58, 73, 49, 39, 28, 1, 80, 0, 67)_{\Theta(1,2,11)},$
 $(80, 83, 37, 43, 2, 56, 8, 33, 84, 69, 14, 67, 38)_{\Theta(1,2,11)},$
 $(9, 71, 75, 28, 38, 7, 79, 64, 19, 53, 6, 13, 63)_{\Theta(1,2,11)},$
 $(30, 12, 6, 43, 25, 16, 66, 9, 29, 64, 46, 68, 84)_{\Theta(1,2,11)},$
 $(19, 81, 69, 21, 74, 16, 11, 82, 38, 78, 89, 10, 73)_{\Theta(1,2,11)},$
 $(3, 43, 71, 10, 88, 4, 27, 2, 86, 69, 50, 73, 75)_{\Theta(1,2,11)},$
 $(22, 1, 55, 50, 45, 17, 78, 82, 81, 25, 58, 42, 15)_{\Theta(1,2,11)},$
 $(69, 64, 11, 48, 84, 81, 21, 73, 44, 13, 89, 35, 34)_{\Theta(1,2,11)},$
 $(39, 80, 23, 83, 24, 58, 72, 33, 29, 88, 22, 49, 12)_{\Theta(1,2,11)},$
 $(65, 72, 76, 14, 26, 31, 40, 8, 48, 70, 59, 81, 78)_{\Theta(1,2,11)},$
 $(39, 22, 13, 40, 17, 23, 14, 52, 24, 43, 86, 84, 58)_{\Theta(1,2,11)},$
 $(16, 14, 46, 6, 49, 74, 19, 42, 23, 41, 51, 64, 71)_{\Theta(1,2,11)},$

$(2, 91, 48, 52, 5, 4, 21, 7, 13, 61, 83, 3, 20)_{\Theta(1,2,11)},$
 $(0, 60, 9, 62, 10, 17, 88, 49, 84, 37, 80, 76, 55)_{\Theta(1,3,10)},$
 $(21, 37, 57, 24, 4, 67, 74, 63, 48, 32, 52, 51, 34)_{\Theta(1,3,10)},$
 $(83, 28, 63, 78, 47, 66, 88, 42, 2, 80, 50, 75, 15)_{\Theta(1,3,10)},$
 $(49, 51, 79, 76, 82, 57, 7, 47, 35, 28, 65, 11, 20)_{\Theta(1,3,10)},$
 $(69, 39, 54, 26, 34, 40, 59, 31, 53, 51, 55, 9, 90)_{\Theta(1,3,10)},$
 $(83, 73, 56, 8, 35, 21, 55, 71, 20, 31, 74, 1, 88)_{\Theta(1,3,10)},$
 $(11, 46, 50, 54, 3, 62, 17, 81, 56, 64, 57, 43, 66)_{\Theta(1,3,10)},$
 $(20, 1, 63, 45, 9, 5, 74, 56, 44, 8, 60, 90, 69)_{\Theta(1,3,10)},$
 $(80, 12, 52, 34, 78, 44, 67, 22, 27, 37, 66, 0, 59)_{\Theta(1,3,10)},$
 $(9, 69, 60, 26, 46, 34, 2, 33, 15, 81, 84, 27, 61)_{\Theta(1,3,10)},$
 $(45, 33, 65, 39, 18, 66, 40, 30, 84, 53, 76, 23, 50)_{\Theta(1,3,10)},$
 $(36, 37, 19, 89, 42, 51, 14, 64, 50, 79, 25, 66, 87)_{\Theta(1,3,10)},$
 $(51, 27, 57, 58, 50, 14, 1, 6, 22, 46, 84, 21, 30)_{\Theta(1,3,10)},$
 $(0, 21, 44, 84, 60, 81, 71, 31, 50, 43, 65, 51, 14)_{\Theta(1,4,9)},$
 $(50, 70, 29, 63, 8, 34, 80, 86, 42, 38, 3, 49, 2)_{\Theta(1,4,9)},$
 $(15, 54, 64, 46, 60, 65, 22, 57, 42, 14, 80, 71, 3)_{\Theta(1,4,9)},$
 $(9, 4, 3, 15, 35, 25, 72, 81, 85, 77, 21, 80, 2)_{\Theta(1,4,9)},$
 $(64, 71, 86, 6, 75, 28, 56, 53, 78, 20, 67, 23, 8)_{\Theta(1,4,9)},$
 $(27, 0, 8, 42, 75, 69, 28, 44, 36, 35, 13, 76, 61)_{\Theta(1,4,9)},$
 $(23, 64, 51, 45, 14, 56, 46, 84, 88, 61, 68, 79, 76)_{\Theta(1,4,9)},$
 $(45, 17, 6, 15, 26, 33, 3, 28, 78, 77, 75, 30, 55)_{\Theta(1,4,9)},$
 $(32, 57, 67, 37, 34, 71, 5, 19, 55, 87, 38, 65, 33)_{\Theta(1,4,9)},$
 $(79, 5, 63, 62, 60, 14, 35, 38, 57, 0, 73, 72, 49)_{\Theta(1,4,9)},$
 $(37, 26, 70, 62, 52, 42, 12, 72, 85, 19, 11, 16, 87)_{\Theta(1,4,9)},$
 $(10, 32, 73, 86, 49, 62, 26, 31, 29, 9, 67, 57, 19)_{\Theta(1,4,9)},$
 $(44, 24, 62, 91, 73, 82, 41, 81, 6, 66, 51, 34, 47)_{\Theta(1,4,9)},$
 $(0, 33, 7, 57, 44, 52, 19, 42, 12, 83, 59, 53, 10)_{\Theta(1,5,8)},$
 $(31, 11, 4, 10, 39, 85, 88, 78, 21, 2, 86, 8, 80)_{\Theta(1,5,8)},$
 $(56, 2, 73, 29, 43, 80, 59, 18, 31, 60, 22, 50, 74)_{\Theta(1,5,8)},$
 $(19, 7, 58, 32, 9, 63, 15, 67, 78, 90, 53, 22, 66)_{\Theta(1,5,8)},$
 $(9, 77, 31, 78, 61, 55, 25, 57, 52, 50, 28, 40, 13)_{\Theta(1,5,8)},$
 $(89, 59, 7, 24, 73, 2, 1, 37, 44, 60, 19, 68, 42)_{\Theta(1,5,8)},$
 $(53, 0, 42, 11, 14, 70, 65, 55, 29, 63, 16, 10, 89)_{\Theta(1,5,8)},$
 $(48, 79, 0, 40, 61, 21, 23, 78, 83, 39, 53, 27, 6)_{\Theta(1,5,8)},$
 $(68, 55, 5, 43, 45, 53, 33, 88, 11, 2, 51, 59, 87)_{\Theta(1,5,8)},$
 $(1, 78, 16, 44, 86, 53, 0, 56, 5, 55, 70, 10, 49)_{\Theta(1,5,8)},$
 $(37, 34, 17, 64, 30, 25, 28, 10, 62, 55, 39, 14, 61)_{\Theta(1,5,8)},$
 $(68, 57, 70, 77, 78, 32, 35, 44, 39, 0, 18, 76, 87)_{\Theta(1,5,8)},$
 $(77, 16, 26, 10, 6, 48, 59, 60, 36, 40, 50, 51, 78)_{\Theta(1,5,8)},$
 $(0, 16, 46, 75, 50, 45, 42, 1, 22, 8, 36, 14, 89)_{\Theta(1,6,7)},$
 $(78, 24, 50, 46, 88, 15, 83, 1, 66, 32, 34, 12, 41)_{\Theta(1,6,7)},$
 $(62, 2, 71, 22, 32, 26, 76, 55, 33, 43, 46, 36, 19)_{\Theta(1,6,7)},$
 $(84, 47, 76, 51, 57, 50, 20, 83, 63, 28, 40, 45, 2)_{\Theta(1,6,7)},$
 $(33, 7, 75, 15, 27, 62, 56, 70, 86, 29, 63, 30, 83)_{\Theta(1,6,7)},$
 $(20, 33, 64, 87, 22, 48, 45, 89, 30, 43, 84, 71, 79)_{\Theta(1,6,7)},$
 $(84, 66, 41, 16, 52, 45, 4, 87, 83, 14, 75, 35, 89)_{\Theta(1,6,7)},$

$(49, 31, 30, 18, 5, 14, 45, 28, 8, 41, 52, 43, 36)_{\Theta(1,6,7)},$
 $(35, 14, 50, 90, 3, 84, 55, 73, 87, 57, 46, 38, 77)_{\Theta(1,6,7)},$
 $(71, 21, 27, 25, 45, 15, 60, 34, 90, 22, 24, 63, 73)_{\Theta(1,6,7)},$
 $(9, 85, 33, 64, 29, 76, 8, 35, 17, 77, 41, 32, 18)_{\Theta(1,6,7)},$
 $(56, 29, 4, 36, 32, 70, 25, 71, 65, 1, 2, 75, 77)_{\Theta(1,6,7)},$
 $(67, 3, 9, 1, 42, 43, 64, 11, 22, 2, 46, 80, 25)_{\Theta(1,6,7)},$
 $(0, 49, 66, 12, 72, 81, 73, 3, 43, 15, 75, 29, 65)_{\Theta(2,3,9)},$
 $(67, 22, 50, 57, 63, 8, 55, 64, 40, 37, 79, 81, 69)_{\Theta(2,3,9)},$
 $(72, 28, 58, 62, 24, 81, 22, 16, 71, 51, 87, 5, 30)_{\Theta(2,3,9)},$
 $(59, 52, 30, 72, 1, 74, 6, 18, 10, 89, 66, 29, 47)_{\Theta(2,3,9)},$
 $(45, 78, 19, 38, 42, 64, 84, 29, 26, 0, 35, 41, 36)_{\Theta(2,3,9)},$
 $(19, 49, 81, 35, 58, 67, 4, 52, 13, 33, 14, 30, 76)_{\Theta(2,3,9)},$
 $(54, 70, 85, 88, 67, 56, 49, 24, 51, 28, 47, 86, 21)_{\Theta(2,3,9)},$
 $(71, 65, 17, 66, 14, 88, 36, 72, 54, 1, 29, 0, 7)_{\Theta(2,3,9)},$
 $(65, 9, 61, 70, 83, 79, 21, 22, 23, 35, 57, 15, 56)_{\Theta(2,3,9)},$
 $(90, 63, 71, 58, 67, 55, 79, 14, 49, 84, 10, 52, 24)_{\Theta(2,3,9)},$
 $(40, 86, 43, 25, 24, 54, 7, 73, 60, 75, 82, 48, 64)_{\Theta(2,3,9)},$
 $(63, 22, 42, 25, 1, 61, 47, 10, 73, 24, 85, 55, 66)_{\Theta(2,3,9)},$
 $(65, 7, 32, 48, 56, 54, 23, 24, 34, 40, 2, 27, 88)_{\Theta(2,3,9)},$
 $(0, 78, 73, 15, 90, 66, 22, 46, 69, 87, 39, 82, 65)_{\Theta(2,5,7)},$
 $(68, 40, 81, 55, 21, 73, 46, 4, 79, 48, 51, 31, 30)_{\Theta(2,5,7)},$
 $(22, 55, 60, 30, 61, 6, 83, 54, 24, 63, 38, 17, 85)_{\Theta(2,5,7)},$
 $(80, 86, 48, 25, 19, 85, 1, 74, 65, 15, 84, 14, 83)_{\Theta(2,5,7)},$
 $(45, 32, 59, 41, 88, 31, 7, 33, 44, 77, 14, 19, 12)_{\Theta(2,5,7)},$
 $(83, 61, 84, 22, 56, 52, 13, 43, 51, 40, 25, 41, 44)_{\Theta(2,5,7)},$
 $(0, 46, 57, 10, 74, 2, 60, 83, 3, 73, 52, 88, 58)_{\Theta(2,5,7)},$
 $(50, 51, 87, 83, 7, 34, 38, 59, 5, 30, 81, 47, 1)_{\Theta(2,5,7)},$
 $(86, 5, 79, 29, 14, 12, 36, 68, 23, 60, 18, 21, 11)_{\Theta(2,5,7)},$
 $(89, 68, 46, 42, 41, 71, 20, 75, 43, 69, 13, 11, 84)_{\Theta(2,5,7)},$
 $(79, 49, 69, 8, 67, 29, 56, 90, 24, 53, 34, 36, 44)_{\Theta(2,5,7)},$
 $(22, 27, 74, 61, 12, 52, 66, 72, 60, 86, 51, 30, 46)_{\Theta(2,5,7)},$
 $(11, 23, 74, 7, 56, 81, 21, 40, 41, 13, 4, 78, 45)_{\Theta(2,5,7)},$
 $(0, 44, 48, 14, 47, 57, 10, 30, 24, 39, 21, 6, 64)_{\Theta(3,3,8)},$
 $(3, 82, 64, 9, 19, 73, 1, 72, 31, 26, 21, 27, 0)_{\Theta(3,3,8)},$
 $(4, 6, 72, 57, 27, 63, 67, 42, 43, 15, 24, 90, 75)_{\Theta(3,3,8)},$
 $(38, 6, 44, 86, 76, 77, 25, 21, 67, 47, 55, 10, 65)_{\Theta(3,3,8)},$
 $(62, 28, 59, 56, 32, 71, 79, 25, 20, 77, 33, 76, 17)_{\Theta(3,3,8)},$
 $(82, 42, 6, 14, 81, 79, 23, 11, 41, 32, 7, 55, 45)_{\Theta(3,3,8)},$
 $(73, 11, 39, 7, 31, 37, 6, 84, 70, 88, 57, 1, 48)_{\Theta(3,3,8)},$
 $(9, 8, 28, 46, 51, 58, 36, 19, 38, 86, 21, 53, 1)_{\Theta(3,3,8)},$
 $(85, 6, 71, 58, 16, 76, 9, 70, 68, 66, 62, 36, 53)_{\Theta(3,3,8)},$
 $(23, 30, 75, 66, 12, 19, 56, 40, 18, 71, 42, 77, 54)_{\Theta(3,3,8)},$
 $(58, 54, 9, 1, 90, 61, 12, 16, 28, 23, 41, 30, 3)_{\Theta(3,3,8)},$
 $(40, 52, 32, 51, 65, 87, 27, 58, 15, 77, 80, 29, 0)_{\Theta(3,3,8)},$
 $(1, 5, 13, 33, 15, 39, 69, 47, 21, 38, 59, 80, 44)_{\Theta(3,3,8)},$
 $(0, 21, 23, 8, 42, 9, 62, 68, 51, 58, 66, 69, 38)_{\Theta(3,4,7)},$
 $(73, 57, 24, 34, 55, 31, 59, 33, 40, 6, 62, 64, 85)_{\Theta(3,4,7)},$

$(85, 65, 60, 64, 38, 3, 68, 83, 50, 67, 22, 1, 21)_{\Theta(3,4,7)},$
 $(5, 56, 81, 38, 30, 19, 11, 62, 44, 63, 33, 9, 43)_{\Theta(3,4,7)},$
 $(5, 82, 87, 34, 31, 45, 59, 6, 27, 33, 62, 63, 22)_{\Theta(3,4,7)},$
 $(42, 52, 82, 86, 40, 24, 90, 67, 55, 48, 28, 14, 8)_{\Theta(3,4,7)},$
 $(67, 22, 19, 34, 17, 75, 3, 21, 10, 25, 37, 76, 90)_{\Theta(3,4,7)},$
 $(27, 8, 76, 79, 52, 51, 81, 82, 56, 3, 59, 85, 77)_{\Theta(3,4,7)},$
 $(27, 88, 36, 48, 14, 11, 61, 56, 41, 46, 26, 31, 13)_{\Theta(3,4,7)},$
 $(58, 48, 27, 70, 86, 3, 79, 31, 68, 26, 64, 75, 16)_{\Theta(3,4,7)},$
 $(43, 52, 8, 44, 47, 85, 80, 33, 65, 46, 68, 13, 17)_{\Theta(3,4,7)},$
 $(21, 9, 84, 74, 57, 16, 46, 14, 60, 30, 36, 87, 47)_{\Theta(3,4,7)},$
 $(90, 7, 14, 1, 27, 32, 77, 81, 48, 17, 8, 89, 67)_{\Theta(3,4,7)},$
 $(0, 66, 12, 68, 89, 24, 33, 22, 81, 90, 43, 67, 36)_{\Theta(3,5,6)},$
 $(51, 6, 45, 35, 71, 24, 73, 85, 53, 75, 43, 16, 1)_{\Theta(3,5,6)},$
 $(41, 35, 33, 1, 64, 40, 21, 88, 48, 68, 87, 4, 74)_{\Theta(3,5,6)},$
 $(88, 63, 34, 5, 6, 25, 76, 48, 39, 69, 52, 22, 40)_{\Theta(3,5,6)},$
 $(15, 64, 44, 48, 4, 70, 39, 82, 66, 29, 28, 72, 77)_{\Theta(3,5,6)},$
 $(3, 49, 90, 82, 20, 80, 23, 42, 30, 34, 10, 77, 50)_{\Theta(3,5,6)},$
 $(72, 45, 38, 76, 79, 39, 75, 31, 37, 85, 42, 40, 7)_{\Theta(3,5,6)},$
 $(51, 47, 63, 38, 37, 22, 86, 44, 18, 3, 78, 55, 29)_{\Theta(3,5,6)},$
 $(21, 54, 90, 4, 5, 43, 80, 34, 25, 45, 47, 69, 51)_{\Theta(3,5,6)},$
 $(89, 61, 38, 26, 7, 3, 2, 33, 27, 6, 46, 20, 19)_{\Theta(3,5,6)},$
 $(59, 21, 51, 77, 66, 82, 85, 61, 22, 5, 76, 55, 80)_{\Theta(3,5,6)},$
 $(0, 14, 84, 74, 79, 15, 2, 72, 40, 85, 39, 44, 50)_{\Theta(3,5,6)},$
 $(59, 42, 70, 56, 2, 73, 44, 81, 8, 61, 55, 39, 89)_{\Theta(3,5,6)},$
 $(0, 2, 4, 88, 6, 82, 38, 8, 83, 26, 65, 77, 30)_{\Theta(4,5,5)},$
 $(70, 66, 45, 21, 16, 38, 71, 46, 26, 72, 42, 28, 74)_{\Theta(4,5,5)},$
 $(49, 15, 11, 73, 42, 36, 83, 24, 1, 82, 4, 77, 13)_{\Theta(4,5,5)},$
 $(82, 48, 8, 20, 19, 41, 9, 57, 37, 33, 14, 68, 17)_{\Theta(4,5,5)},$
 $(89, 20, 56, 40, 75, 18, 9, 0, 54, 39, 8, 65, 64)_{\Theta(4,5,5)},$
 $(6, 43, 3, 37, 22, 69, 79, 23, 15, 5, 68, 12, 19)_{\Theta(4,5,5)},$
 $(86, 10, 7, 48, 87, 35, 32, 53, 17, 69, 4, 26, 8)_{\Theta(4,5,5)},$
 $(84, 4, 41, 48, 75, 35, 66, 23, 32, 79, 62, 78, 10)_{\Theta(4,5,5)},$
 $(20, 60, 72, 17, 83, 5, 31, 78, 84, 45, 0, 53, 26)_{\Theta(4,5,5)},$
 $(70, 1, 82, 83, 9, 0, 79, 63, 6, 75, 85, 55, 23)_{\Theta(4,5,5)},$
 $(58, 76, 35, 49, 87, 19, 41, 1, 51, 16, 88, 15, 73)_{\Theta(4,5,5)},$
 $(56, 15, 88, 13, 89, 71, 31, 79, 6, 30, 33, 29, 35)_{\Theta(4,5,5)},$
 $(19, 3, 7, 9, 66, 23, 78, 65, 10, 25, 2, 38, 49)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 4 \pmod{92}$. □

Proof of Lemma 3.10

$K_{14,7}$ Let the vertex set be $\{0, 1, \dots, 20\}$ partitioned into $\{0, 1, \dots, 13\}$ and $\{14, 15, \dots, 20\}$. The decompositions consist of

$(0, 1, 14, 16, 15, 4, 19, 2, 20, 11, 18, 13, 17)_{\Theta(2,2,10)},$
 $(0, 9, 15, 16, 8, 17, 14, 6, 20, 13, 18, 5, 19)_{\Theta(2,4,8)},$
 $(0, 9, 18, 15, 4, 16, 13, 19, 17, 10, 14, 5, 20)_{\Theta(2,6,6)},$
 $(0, 11, 17, 4, 18, 19, 10, 16, 20, 7, 15, 1, 14)_{\Theta(4,4,6)}$

under the action of the mapping $x \mapsto x + 2 \pmod{14}$ for $x < 14$, $x \mapsto (x + 1 \pmod{7}) + 14$ for $x \geq 14$.

$K_{14,14,14}$ Let the vertex set be Z_{42} partitioned according to residue classes modulo 3. The decompositions consist of

$$\begin{aligned}
& (0, 17, 16, 10, 18, 11, 30, 34, 3, 1, 2, 4, 9)_{\Theta(1,2,11)}, \\
& (0, 7, 20, 13, 3, 8, 22, 5, 1, 12, 31, 9, 23)_{\Theta(1,2,11)}, \\
& (0, 20, 17, 39, 37, 27, 13, 8, 1, 2, 3, 5, 9)_{\Theta(1,3,10)}, \\
& (0, 2, 4, 12, 7, 15, 31, 6, 19, 8, 22, 9, 28)_{\Theta(1,3,10)}, \\
& (0, 14, 10, 27, 25, 34, 18, 7, 2, 1, 5, 4, 33)_{\Theta(1,4,9)}, \\
& (0, 2, 4, 11, 25, 13, 3, 8, 1, 9, 31, 5, 22)_{\Theta(1,4,9)}, \\
& (0, 29, 10, 9, 23, 21, 1, 11, 27, 5, 40, 26, 4)_{\Theta(1,5,8)}, \\
& (21, 32, 10, 2, 19, 6, 16, 11, 15, 22, 3, 26, 28)_{\Theta(1,5,8)}, \\
& (0, 19, 16, 29, 34, 32, 39, 35, 12, 37, 6, 17, 18)_{\Theta(1,6,7)}, \\
& (13, 8, 9, 17, 7, 20, 0, 15, 29, 3, 28, 14, 4)_{\Theta(1,6,7)}, \\
& (0, 23, 28, 17, 31, 5, 34, 27, 26, 6, 38, 30, 13)_{\Theta(2,3,9)}, \\
& (28, 13, 17, 30, 14, 15, 4, 0, 7, 5, 25, 6, 29)_{\Theta(2,3,9)}, \\
& (0, 12, 40, 31, 14, 27, 8, 34, 18, 19, 32, 15, 35)_{\Theta(2,5,7)}, \\
& (29, 33, 25, 13, 18, 38, 1, 15, 8, 7, 5, 12, 22)_{\Theta(2,5,7)}, \\
& (0, 23, 25, 27, 40, 33, 1, 12, 31, 15, 34, 29, 9)_{\Theta(3,3,8)}, \\
& (28, 39, 20, 31, 32, 22, 0, 7, 2, 15, 16, 36, 10)_{\Theta(3,3,8)}, \\
& (0, 3, 1, 20, 28, 12, 23, 29, 30, 5, 10, 32, 37)_{\Theta(3,4,7)}, \\
& (39, 34, 29, 36, 23, 25, 11, 1, 12, 2, 6, 13, 0)_{\Theta(3,4,7)}, \\
& (0, 2, 37, 36, 23, 30, 8, 40, 13, 11, 12, 28, 33)_{\Theta(3,5,6)}, \\
& (31, 13, 5, 27, 14, 0, 7, 3, 2, 4, 15, 38, 21)_{\Theta(3,5,6)}, \\
& (0, 23, 8, 25, 24, 11, 22, 29, 39, 37, 32, 12, 10)_{\Theta(4,5,5)}, \\
& (40, 39, 27, 35, 16, 8, 4, 18, 1, 14, 7, 5, 19)_{\Theta(4,5,5)}
\end{aligned}$$

under the action of the mapping $x \mapsto x + 2 \pmod{42}$.

$K_{7,7,7,7}$ Let the vertex set be $\{0, 1, \dots, 27\}$ partitioned into $\{3j + i : j = 0, 1, \dots, 6\}, i = 0, 1, 2,$ and $\{21, 22, \dots, 27\}$. The decompositions consist of

$$\begin{aligned}
& (0, 23, 13, 7, 17, 1, 2, 21, 3, 5, 9, 22, 8)_{\Theta(1,2,11)}, \\
& (0, 25, 1, 11, 2, 21, 12, 8, 13, 6, 14, 22, 5)_{\Theta(1,3,10)}, \\
& (0, 24, 14, 18, 19, 13, 11, 26, 1, 6, 16, 22, 8)_{\Theta(1,4,9)}, \\
& (0, 23, 1, 21, 6, 16, 17, 3, 8, 25, 7, 5, 18)_{\Theta(1,5,8)}, \\
& (0, 22, 26, 17, 1, 18, 4, 21, 8, 6, 7, 15, 5)_{\Theta(1,6,7)}, \\
& (0, 21, 13, 5, 9, 19, 20, 26, 2, 12, 23, 7, 14)_{\Theta(2,3,9)}, \\
& (0, 22, 17, 11, 10, 12, 4, 5, 19, 21, 6, 23, 2)_{\Theta(2,5,7)}, \\
& (0, 21, 26, 20, 11, 18, 19, 3, 2, 6, 24, 1, 14)_{\Theta(3,3,8)}, \\
& (0, 21, 10, 12, 24, 3, 17, 16, 8, 4, 5, 25, 6)_{\Theta(3,4,7)}, \\
& (0, 23, 24, 2, 10, 11, 15, 7, 14, 9, 22, 3, 5)_{\Theta(3,5,6)}, \\
& (0, 25, 4, 15, 1, 23, 8, 21, 7, 2, 3, 11, 6)_{\Theta(4,5,5)}
\end{aligned}$$

under the action of the mapping $x \mapsto x + 1 \pmod{21}$ for $x < 21$, $x \mapsto (x + 1 \pmod{7}) + 21$ for

$x \geq 21$.

$K_{28,28,28,35}$ Let the vertex set be $\{0, 1, \dots, 118\}$ partitioned into $\{3j + i : j = 0, 1, \dots, 27\}$, $i = 0, 1, 2$, and $\{84, 85, \dots, 118\}$. The decompositions consist of

- $(0, 87, 65, 2, 113, 20, 36, 84, 74, 101, 12, 52, 11)_{\Theta(1,2,11)},$
- $(45, 26, 100, 20, 25, 83, 61, 54, 87, 44, 81, 116, 19)_{\Theta(1,2,11)},$
- $(35, 6, 93, 18, 106, 30, 90, 66, 40, 17, 84, 19, 62)_{\Theta(1,2,11)},$
- $(79, 29, 116, 35, 40, 41, 39, 31, 3, 99, 49, 69, 98)_{\Theta(1,2,11)},$
- $(75, 111, 71, 117, 25, 15, 50, 1, 108, 72, 99, 29, 67)_{\Theta(1,2,11)},$
- $(24, 2, 94, 32, 0, 10, 81, 65, 95, 11, 90, 20, 6)_{\Theta(1,2,11)},$
- $(69, 80, 16, 117, 44, 93, 4, 15, 90, 33, 103, 21, 92)_{\Theta(1,2,11)},$
- $(112, 55, 72, 20, 67, 108, 76, 75, 43, 85, 68, 34, 99)_{\Theta(1,2,11)},$
- $(116, 68, 30, 28, 41, 10, 69, 46, 17, 3, 94, 6, 110)_{\Theta(1,2,11)},$
- $(0, 25, 86, 81, 73, 65, 60, 111, 12, 38, 109, 27, 50)_{\Theta(1,3,10)},$
- $(73, 90, 75, 5, 88, 49, 59, 55, 6, 29, 82, 80, 9)_{\Theta(1,3,10)},$
- $(86, 76, 43, 32, 73, 78, 11, 88, 10, 63, 115, 52, 94)_{\Theta(1,3,10)},$
- $(0, 74, 89, 45, 92, 6, 71, 72, 79, 95, 28, 69, 88)_{\Theta(1,3,10)},$
- $(13, 71, 53, 113, 68, 40, 24, 84, 55, 5, 70, 23, 102)_{\Theta(1,3,10)},$
- $(77, 110, 102, 44, 0, 4, 12, 13, 85, 18, 117, 53, 66)_{\Theta(1,3,10)},$
- $(111, 73, 33, 104, 37, 117, 76, 95, 82, 32, 67, 94, 51)_{\Theta(1,3,10)},$
- $(27, 7, 79, 92, 11, 101, 24, 10, 108, 78, 102, 62, 45)_{\Theta(1,3,10)},$
- $(15, 52, 103, 74, 56, 4, 106, 6, 26, 64, 88, 20, 89)_{\Theta(1,3,10)},$
- $(0, 38, 19, 78, 95, 77, 109, 80, 63, 102, 25, 36, 70)_{\Theta(1,4,9)},$
- $(10, 66, 97, 48, 62, 104, 39, 11, 9, 113, 8, 30, 85)_{\Theta(1,4,9)},$
- $(17, 96, 27, 79, 80, 9, 35, 70, 112, 60, 73, 56, 30)_{\Theta(1,4,9)},$
- $(68, 76, 63, 111, 15, 25, 109, 6, 105, 31, 95, 57, 101)_{\Theta(1,4,9)},$
- $(0, 89, 88, 76, 77, 65, 36, 83, 97, 16, 23, 46, 57)_{\Theta(1,4,9)},$
- $(116, 69, 42, 104, 1, 8, 10, 50, 70, 23, 91, 61, 102)_{\Theta(1,4,9)},$
- $(60, 84, 110, 11, 36, 107, 22, 57, 35, 81, 76, 93, 79)_{\Theta(1,4,9)},$
- $(19, 39, 113, 60, 70, 15, 105, 8, 86, 32, 3, 73, 88)_{\Theta(1,4,9)},$
- $(30, 73, 46, 77, 93, 88, 66, 53, 13, 63, 95, 9, 102)_{\Theta(1,4,9)},$
- $(0, 19, 91, 23, 93, 62, 70, 96, 7, 51, 40, 78, 112)_{\Theta(1,5,8)},$
- $(68, 66, 96, 50, 75, 14, 40, 53, 51, 55, 85, 76, 90)_{\Theta(1,5,8)},$
- $(114, 72, 36, 1, 15, 5, 75, 70, 98, 54, 84, 17, 117)_{\Theta(1,5,8)},$
- $(16, 38, 90, 59, 89, 64, 11, 87, 6, 103, 68, 55, 27)_{\Theta(1,5,8)},$
- $(75, 16, 110, 82, 113, 23, 102, 8, 58, 3, 87, 1, 111)_{\Theta(1,5,8)},$
- $(1, 8, 105, 27, 11, 61, 23, 86, 53, 79, 15, 91, 45)_{\Theta(1,5,8)},$
- $(31, 39, 103, 46, 45, 93, 100, 44, 91, 32, 75, 28, 8)_{\Theta(1,5,8)},$
- $(47, 1, 88, 31, 80, 115, 104, 69, 14, 30, 40, 0, 99)_{\Theta(1,5,8)},$
- $(38, 104, 61, 9, 26, 7, 34, 35, 87, 0, 76, 117, 8)_{\Theta(1,5,8)},$
- $(0, 80, 62, 42, 100, 79, 12, 40, 71, 96, 41, 85, 66)_{\Theta(1,6,7)},$
- $(69, 68, 90, 30, 107, 15, 43, 11, 76, 89, 73, 77, 106)_{\Theta(1,6,7)},$
- $(81, 32, 56, 103, 62, 6, 87, 4, 109, 76, 90, 78, 89)_{\Theta(1,6,7)},$
- $(63, 2, 94, 74, 116, 61, 86, 43, 59, 13, 14, 67, 87)_{\Theta(1,6,7)},$
- $(6, 64, 77, 12, 91, 24, 103, 113, 15, 4, 59, 92, 30)_{\Theta(1,6,7)},$
- $(58, 66, 96, 5, 113, 17, 115, 9, 70, 59, 15, 85, 79)_{\Theta(1,6,7)},$
- $(14, 4, 9, 76, 107, 17, 103, 66, 68, 92, 5, 109, 50)_{\Theta(1,6,7)},$

$(64, 69, 27, 20, 49, 41, 7, 93, 43, 112, 79, 38, 111)_{\Theta(1,6,7)},$
 $(18, 55, 61, 71, 73, 59, 99, 90, 9, 108, 11, 63, 104)_{\Theta(1,6,7)},$
 $(0, 94, 56, 55, 27, 5, 105, 14, 1, 50, 28, 111, 35)_{\Theta(2,3,9)},$
 $(111, 114, 66, 51, 37, 72, 1, 48, 73, 6, 79, 32, 70)_{\Theta(2,3,9)},$
 $(106, 111, 20, 52, 11, 69, 92, 50, 58, 62, 102, 4, 27)_{\Theta(2,3,9)},$
 $(74, 76, 110, 31, 103, 39, 55, 15, 53, 19, 88, 32, 42)_{\Theta(2,3,9)},$
 $(83, 116, 19, 61, 62, 1, 24, 64, 9, 13, 78, 76, 77)_{\Theta(2,3,9)},$
 $(36, 102, 56, 97, 47, 10, 27, 19, 100, 78, 46, 39, 44)_{\Theta(2,3,9)},$
 $(36, 49, 89, 105, 17, 94, 32, 113, 11, 95, 75, 112, 68)_{\Theta(2,3,9)},$
 $(80, 81, 108, 10, 98, 49, 23, 109, 74, 6, 13, 99, 7)_{\Theta(2,3,9)},$
 $(117, 54, 29, 33, 85, 59, 108, 4, 35, 24, 118, 11, 105)_{\Theta(2,3,9)},$
 $(0, 39, 7, 74, 49, 93, 23, 25, 95, 29, 34, 108, 28)_{\Theta(2,5,7)},$
 $(9, 105, 55, 92, 83, 109, 76, 91, 77, 19, 96, 8, 7)_{\Theta(2,5,7)},$
 $(34, 33, 98, 107, 4, 65, 52, 32, 25, 91, 55, 86, 31)_{\Theta(2,5,7)},$
 $(82, 76, 27, 117, 48, 97, 33, 14, 6, 73, 63, 77, 54)_{\Theta(2,5,7)},$
 $(100, 36, 7, 68, 54, 107, 70, 37, 15, 55, 51, 85, 10)_{\Theta(2,5,7)},$
 $(47, 22, 108, 30, 94, 82, 42, 55, 21, 89, 69, 68, 111)_{\Theta(2,5,7)},$
 $(55, 70, 35, 94, 60, 84, 57, 83, 46, 0, 47, 42, 38)_{\Theta(2,5,7)},$
 $(64, 60, 116, 23, 87, 5, 114, 11, 117, 57, 103, 24, 111)_{\Theta(2,5,7)},$
 $(6, 32, 90, 79, 109, 68, 95, 34, 3, 98, 0, 103, 51)_{\Theta(2,5,7)},$
 $(0, 108, 16, 21, 62, 58, 91, 12, 11, 84, 10, 88, 6)_{\Theta(3,3,8)},$
 $(42, 25, 102, 69, 109, 74, 43, 41, 104, 70, 17, 39, 71)_{\Theta(3,3,8)},$
 $(35, 21, 103, 13, 86, 79, 94, 30, 20, 100, 14, 6, 52)_{\Theta(3,3,8)},$
 $(62, 56, 60, 107, 49, 33, 73, 18, 116, 53, 43, 80, 63)_{\Theta(3,3,8)},$
 $(60, 96, 10, 75, 5, 12, 32, 69, 50, 76, 105, 54, 77)_{\Theta(3,3,8)},$
 $(93, 32, 7, 91, 39, 102, 60, 17, 28, 8, 67, 72, 87)_{\Theta(3,3,8)},$
 $(86, 60, 42, 105, 39, 89, 46, 98, 1, 95, 45, 25, 8)_{\Theta(3,3,8)},$
 $(44, 13, 9, 100, 89, 17, 58, 83, 49, 84, 41, 87, 0)_{\Theta(3,3,8)},$
 $(55, 56, 69, 115, 27, 113, 92, 5, 85, 9, 107, 59, 16)_{\Theta(3,3,8)},$
 $(0, 72, 84, 44, 95, 19, 65, 22, 63, 61, 27, 58, 32)_{\Theta(3,4,7)},$
 $(24, 70, 19, 109, 117, 57, 32, 84, 49, 69, 50, 30, 86)_{\Theta(3,4,7)},$
 $(89, 79, 28, 80, 73, 83, 109, 35, 4, 75, 71, 88, 56)_{\Theta(3,4,7)},$
 $(30, 94, 59, 43, 5, 115, 40, 31, 53, 27, 116, 72, 38)_{\Theta(3,4,7)},$
 $(64, 17, 68, 45, 110, 2, 112, 111, 9, 113, 43, 51, 97)_{\Theta(3,4,7)},$
 $(50, 15, 31, 103, 113, 42, 26, 33, 80, 100, 37, 96, 10)_{\Theta(3,4,7)},$
 $(72, 30, 79, 65, 70, 33, 20, 58, 50, 117, 62, 27, 85)_{\Theta(3,4,7)},$
 $(52, 29, 23, 12, 102, 76, 97, 88, 16, 113, 3, 106, 81)_{\Theta(3,4,7)},$
 $(48, 43, 7, 110, 115, 49, 102, 111, 4, 113, 13, 53, 96)_{\Theta(3,4,7)},$
 $(0, 34, 26, 33, 87, 43, 110, 24, 95, 83, 1, 114, 62)_{\Theta(3,5,6)},$
 $(78, 71, 64, 88, 5, 93, 44, 48, 58, 3, 22, 98, 34)_{\Theta(3,5,6)},$
 $(45, 105, 53, 25, 1, 68, 104, 47, 76, 81, 8, 49, 66)_{\Theta(3,5,6)},$
 $(60, 91, 111, 62, 55, 85, 78, 13, 68, 89, 67, 102, 19)_{\Theta(3,5,6)},$
 $(78, 36, 44, 112, 65, 117, 20, 106, 80, 98, 51, 109, 1)_{\Theta(3,5,6)},$
 $(90, 111, 37, 57, 8, 63, 5, 75, 70, 114, 61, 48, 25)_{\Theta(3,5,6)},$
 $(54, 69, 76, 17, 13, 112, 14, 93, 29, 39, 46, 92, 65)_{\Theta(3,5,6)},$
 $(13, 70, 88, 38, 51, 50, 95, 33, 75, 101, 36, 20, 104)_{\Theta(3,5,6)},$

$(38, 59, 78, 25, 96, 67, 109, 24, 0, 97, 13, 29, 108)_{\Theta(3,5,6)},$
 $(0, 38, 61, 86, 58, 76, 47, 89, 42, 84, 36, 79, 100)_{\Theta(4,5,5)},$
 $(62, 18, 100, 54, 101, 79, 115, 13, 107, 76, 42, 29, 43)_{\Theta(4,5,5)},$
 $(31, 30, 51, 32, 84, 78, 103, 66, 65, 106, 46, 69, 91)_{\Theta(4,5,5)},$
 $(109, 58, 76, 5, 103, 31, 102, 33, 97, 61, 23, 16, 27)_{\Theta(4,5,5)},$
 $(99, 32, 5, 22, 97, 77, 88, 57, 76, 31, 27, 104, 70)_{\Theta(4,5,5)},$
 $(25, 19, 96, 12, 44, 88, 40, 103, 81, 9, 17, 108, 53)_{\Theta(4,5,5)},$
 $(12, 54, 68, 13, 112, 70, 111, 27, 95, 53, 48, 105, 32)_{\Theta(4,5,5)},$
 $(46, 38, 77, 67, 92, 98, 82, 102, 27, 47, 0, 49, 54)_{\Theta(4,5,5)},$
 $(39, 71, 116, 83, 27, 115, 75, 17, 19, 46, 44, 54, 110)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 2 \pmod{84}$ for $x < 84$, $x \mapsto (x - 84 + 5 \pmod{35}) + 84$ for $x \geq 84$.

$K_{7,7,7,7}$ Let the vertex set be Z_{35} partitioned according to residue class modulo 5. The decompositions consist of

$(0, 9, 23, 3, 16, 24, 18, 20, 1, 8, 4, 21, 10)_{\Theta(1,2,11)},$
 $(0, 32, 1, 3, 22, 29, 6, 33, 2, 16, 5, 24, 15)_{\Theta(1,3,10)},$
 $(0, 9, 18, 12, 20, 13, 14, 28, 5, 1, 3, 19, 16)_{\Theta(1,4,9)},$
 $(0, 22, 14, 18, 27, 11, 33, 6, 3, 4, 10, 17, 5)_{\Theta(1,5,8)},$
 $(0, 23, 13, 15, 31, 3, 14, 1, 7, 4, 18, 10, 6)_{\Theta(1,6,7)},$
 $(0, 2, 29, 4, 21, 28, 15, 17, 3, 6, 5, 14, 25)_{\Theta(2,3,9)},$
 $(0, 26, 18, 7, 4, 33, 14, 13, 2, 1, 5, 3, 12)_{\Theta(2,5,7)},$
 $(0, 14, 9, 12, 16, 3, 27, 10, 4, 5, 17, 21, 7)_{\Theta(3,3,8)},$
 $(0, 12, 27, 9, 23, 7, 16, 14, 1, 2, 13, 11, 5)_{\Theta(3,4,7)},$
 $(0, 5, 6, 22, 26, 3, 11, 18, 1, 4, 28, 7, 9)_{\Theta(3,5,6)},$
 $(0, 16, 12, 14, 17, 7, 29, 2, 20, 6, 22, 1, 25)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 1 \pmod{35}$.

$K_{7,7,7,21}$ Let the vertex set be $\{0, 1, \dots, 48\}$ partitioned into $\{3j + i : j = 0, 1, \dots, 6\}$, $i = 0, 1, 2$, $\{21, 22, \dots, 27\}$ and $\{28, 29, \dots, 48\}$. The decompositions consist of the graphs

$(0, 4, 5, 35, 14, 30, 15, 21, 39, 22, 45, 1, 25)_{\Theta(1,2,11)},$
 $(42, 17, 10, 16, 8, 19, 32, 3, 44, 22, 36, 9, 25)_{\Theta(1,2,11)},$
 $(18, 30, 20, 40, 21, 16, 27, 33, 9, 28, 10, 38, 0)_{\Theta(1,2,11)},$
 $(0, 32, 25, 20, 34, 7, 36, 23, 14, 22, 40, 17, 12)_{\Theta(1,3,10)},$
 $(18, 11, 21, 15, 5, 45, 2, 30, 14, 44, 23, 32, 1)_{\Theta(1,3,10)},$
 $(32, 15, 17, 41, 11, 25, 42, 3, 1, 0, 46, 24, 29)_{\Theta(1,3,10)},$
 $(0, 4, 10, 45, 3, 21, 18, 43, 13, 5, 23, 37, 6)_{\Theta(1,4,9)},$
 $(21, 43, 32, 20, 15, 44, 4, 30, 19, 34, 0, 45, 2)_{\Theta(1,4,9)},$
 $(45, 26, 21, 2, 39, 1, 24, 4, 31, 13, 6, 35, 18)_{\Theta(1,4,9)},$
 $(0, 19, 26, 5, 13, 34, 31, 17, 25, 46, 8, 3, 35)_{\Theta(1,5,8)},$
 $(27, 33, 10, 36, 24, 8, 31, 1, 26, 35, 7, 30, 11)_{\Theta(1,5,8)},$
 $(30, 22, 18, 7, 0, 39, 6, 10, 11, 31, 2, 29, 16)_{\Theta(1,5,8)},$
 $(0, 34, 20, 1, 26, 28, 3, 43, 6, 19, 30, 9, 24)_{\Theta(1,6,7)},$
 $(13, 32, 22, 35, 27, 45, 6, 9, 47, 26, 19, 42, 18)_{\Theta(1,6,7)},$

$(8, 13, 38, 2, 31, 19, 25, 1, 12, 22, 34, 6, 33)_{\Theta(1,6,7)},$
 $(0, 30, 7, 47, 16, 21, 46, 1, 40, 26, 13, 25, 8)_{\Theta(2,3,9)},$
 $(6, 17, 34, 46, 9, 47, 24, 32, 5, 3, 7, 37, 16)_{\Theta(2,3,9)},$
 $(31, 17, 26, 19, 32, 21, 10, 0, 16, 41, 9, 38, 25)_{\Theta(2,3,9)},$
 $(0, 18, 39, 41, 8, 23, 43, 5, 40, 26, 45, 22, 20)_{\Theta(2,5,7)},$
 $(3, 38, 14, 21, 32, 6, 10, 27, 35, 1, 45, 15, 16)_{\Theta(2,5,7)},$
 $(41, 0, 14, 10, 46, 8, 40, 4, 17, 22, 1, 30, 27)_{\Theta(2,5,7)},$
 $(0, 6, 24, 4, 36, 14, 42, 19, 45, 26, 1, 18, 30)_{\Theta(3,3,8)},$
 $(0, 20, 26, 29, 22, 37, 5, 40, 15, 31, 13, 12, 39)_{\Theta(3,3,8)},$
 $(1, 3, 35, 24, 11, 31, 15, 44, 12, 43, 22, 28, 26)_{\Theta(3,3,8)},$
 $(0, 41, 20, 6, 39, 5, 1, 32, 4, 47, 11, 42, 17)_{\Theta(3,4,7)},$
 $(40, 11, 13, 23, 22, 44, 25, 27, 36, 3, 33, 12, 28)_{\Theta(3,4,7)},$
 $(43, 31, 2, 24, 26, 6, 8, 14, 1, 12, 7, 23, 5)_{\Theta(3,4,7)},$
 $(0, 1, 5, 26, 40, 22, 29, 2, 24, 37, 7, 43, 15)_{\Theta(3,5,6)},$
 $(1, 42, 5, 25, 14, 34, 11, 27, 44, 9, 38, 20, 18)_{\Theta(3,5,6)},$
 $(10, 34, 22, 0, 36, 27, 46, 9, 31, 14, 4, 35, 2)_{\Theta(3,5,6)},$
 $(0, 43, 28, 16, 2, 42, 4, 38, 19, 39, 3, 20, 22)_{\Theta(4,5,5)},$
 $(20, 12, 9, 44, 26, 7, 24, 19, 17, 45, 21, 29, 23)_{\Theta(4,5,5)},$
 $(41, 17, 10, 47, 18, 15, 45, 1, 23, 9, 36, 27, 39)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 1 \pmod{21}$ for $x < 21$, $x \mapsto (x + 1 \pmod{7}) + 21$ for $21 \leq x < 28$, $x \mapsto (x - 28 + 1 \pmod{21}) + 28$ for $x \geq 28$.

$K_{7,7,7,7,28}$ Let the vertex set be $\{0, 1, \dots, 55\}$ partitioned into $\{3j + i : j = 0, 1, \dots, 6\}$, $i = 0, 1, 2, 3$ and $\{28, 29, \dots, 55\}$. The decompositions consist of the graphs

$(0, 18, 35, 6, 43, 4, 28, 5, 40, 27, 34, 16, 42)_{\Theta(1,2,11)},$
 $(21, 47, 23, 16, 54, 25, 12, 5, 27, 29, 0, 42, 6)_{\Theta(1,2,11)},$
 $(16, 44, 18, 27, 49, 22, 51, 19, 9, 14, 1, 7, 10)_{\Theta(1,2,11)},$
 $(38, 17, 6, 21, 30, 3, 51, 23, 42, 27, 14, 16, 49)_{\Theta(1,2,11)},$
 $(4, 11, 47, 35, 8, 3, 42, 9, 36, 25, 18, 17, 51)_{\Theta(1,2,11)},$
 $(37, 2, 11, 16, 32, 27, 33, 18, 36, 17, 41, 24, 46)_{\Theta(1,2,11)},$
 $(21, 34, 11, 29, 17, 37, 27, 48, 26, 38, 8, 28, 3)_{\Theta(1,2,11)},$
 $(1, 47, 0, 12, 26, 29, 13, 43, 27, 44, 8, 45, 22)_{\Theta(1,2,11)},$
 $(28, 26, 16, 18, 7, 4, 45, 20, 54, 1, 51, 0, 27)_{\Theta(1,2,11)},$
 $(13, 10, 16, 44, 15, 52, 9, 7, 49, 2, 50, 22, 43)_{\Theta(1,2,11)},$
 $(49, 10, 3, 21, 7, 16, 48, 6, 11, 24, 5, 43, 1)_{\Theta(1,2,11)},$
 $(0, 44, 42, 25, 2, 8, 45, 22, 16, 23, 28, 10, 3)_{\Theta(1,3,10)},$
 $(13, 49, 10, 19, 39, 21, 28, 0, 32, 12, 47, 18, 20)_{\Theta(1,3,10)},$
 $(52, 23, 3, 26, 1, 8, 39, 22, 34, 4, 37, 2, 9)_{\Theta(1,3,10)},$
 $(15, 40, 46, 14, 24, 6, 44, 1, 10, 49, 8, 23, 0)_{\Theta(1,3,10)},$
 $(35, 2, 25, 39, 22, 28, 1, 42, 8, 34, 19, 20, 30)_{\Theta(1,3,10)},$
 $(1, 41, 18, 19, 51, 27, 17, 12, 34, 26, 45, 13, 3)_{\Theta(1,3,10)},$
 $(47, 19, 13, 42, 11, 26, 29, 3, 1, 12, 9, 0, 53)_{\Theta(1,3,10)},$
 $(30, 5, 9, 29, 19, 33, 12, 23, 32, 10, 37, 22, 11)_{\Theta(1,3,10)},$
 $(8, 26, 53, 25, 9, 24, 43, 27, 1, 32, 18, 5, 42)_{\Theta(1,3,10)},$
 $(34, 10, 15, 35, 1, 31, 19, 37, 21, 32, 24, 27, 5)_{\Theta(1,3,10)},$
 $(28, 4, 26, 47, 11, 38, 3, 35, 8, 54, 6, 20, 43)_{\Theta(1,3,10)},$

$(0, 32, 28, 26, 25, 40, 22, 39, 13, 46, 23, 29, 16)_{\Theta(1,4,9)},$
 $(50, 6, 22, 30, 3, 1, 43, 12, 37, 5, 48, 21, 8)_{\Theta(1,4,9)},$
 $(0, 23, 10, 32, 22, 42, 18, 33, 16, 36, 3, 51, 17)_{\Theta(1,4,9)},$
 $(19, 0, 43, 3, 52, 34, 2, 49, 26, 40, 11, 12, 6)_{\Theta(1,4,9)},$
 $(27, 24, 44, 8, 26, 16, 42, 4, 11, 36, 10, 23, 51)_{\Theta(1,4,9)},$
 $(0, 47, 21, 38, 8, 33, 2, 16, 25, 36, 27, 49, 3)_{\Theta(1,4,9)},$
 $(31, 10, 22, 49, 13, 21, 39, 6, 1, 52, 18, 30, 17)_{\Theta(1,4,9)},$
 $(28, 9, 15, 54, 4, 25, 0, 43, 8, 42, 7, 38, 18)_{\Theta(1,4,9)},$
 $(50, 21, 25, 27, 10, 3, 33, 13, 43, 18, 23, 14, 7)_{\Theta(1,4,9)},$
 $(37, 25, 11, 47, 24, 13, 41, 20, 38, 1, 12, 49, 7)_{\Theta(1,4,9)},$
 $(26, 33, 37, 8, 23, 13, 3, 5, 55, 14, 43, 11, 17)_{\Theta(1,4,9)},$
 $(0, 48, 53, 6, 30, 22, 41, 3, 39, 4, 14, 24, 18)_{\Theta(1,5,8)},$
 $(53, 20, 3, 54, 16, 44, 22, 43, 10, 25, 23, 14, 39)_{\Theta(1,5,8)},$
 $(17, 18, 33, 4, 21, 32, 44, 10, 1, 49, 14, 27, 28)_{\Theta(1,5,8)},$
 $(11, 25, 48, 16, 7, 0, 39, 15, 41, 20, 18, 23, 2)_{\Theta(1,5,8)},$
 $(50, 7, 6, 46, 11, 5, 23, 12, 42, 0, 43, 9, 37)_{\Theta(1,5,8)},$
 $(50, 5, 9, 49, 12, 43, 18, 45, 22, 51, 24, 10, 28)_{\Theta(1,5,8)},$
 $(23, 0, 28, 7, 41, 2, 20, 43, 27, 5, 48, 3, 50)_{\Theta(1,5,8)},$
 $(32, 1, 10, 17, 47, 8, 7, 6, 3, 42, 22, 35, 26)_{\Theta(1,5,8)},$
 $(51, 25, 19, 31, 9, 44, 5, 22, 11, 53, 10, 4, 30)_{\Theta(1,5,8)},$
 $(51, 9, 3, 16, 25, 24, 6, 34, 0, 36, 20, 32, 19)_{\Theta(1,5,8)},$
 $(25, 49, 50, 4, 3, 13, 54, 5, 42, 11, 29, 12, 17)_{\Theta(1,5,8)},$
 $(0, 31, 46, 17, 42, 5, 15, 35, 9, 27, 41, 21, 4)_{\Theta(1,6,7)},$
 $(48, 0, 23, 38, 22, 3, 37, 24, 2, 42, 1, 50, 5)_{\Theta(1,6,7)},$
 $(2, 39, 5, 3, 40, 10, 0, 29, 17, 4, 30, 19, 18)_{\Theta(1,6,7)},$
 $(1, 22, 36, 25, 0, 32, 9, 18, 17, 39, 20, 48, 19)_{\Theta(1,6,7)},$
 $(0, 26, 41, 17, 23, 54, 19, 40, 2, 47, 9, 51, 21)_{\Theta(1,6,7)},$
 $(19, 39, 43, 0, 15, 37, 11, 32, 18, 52, 2, 49, 3)_{\Theta(1,6,7)},$
 $(11, 28, 12, 46, 14, 37, 1, 30, 20, 42, 18, 9, 23)_{\Theta(1,6,7)},$
 $(54, 21, 6, 37, 16, 9, 52, 24, 18, 33, 17, 49, 20)_{\Theta(1,6,7)},$
 $(1, 3, 35, 22, 29, 27, 20, 47, 15, 12, 2, 41, 8)_{\Theta(1,6,7)},$
 $(51, 18, 22, 48, 12, 28, 7, 11, 26, 54, 3, 41, 16)_{\Theta(1,6,7)},$
 $(38, 2, 11, 6, 20, 1, 48, 24, 15, 21, 49, 4, 55)_{\Theta(1,6,7)},$
 $(0, 30, 10, 48, 17, 50, 23, 12, 52, 22, 37, 18, 1)_{\Theta(2,3,9)},$
 $(11, 37, 5, 42, 17, 49, 13, 52, 24, 18, 45, 12, 9)_{\Theta(2,3,9)},$
 $(11, 0, 46, 37, 13, 29, 12, 1, 6, 45, 8, 49, 19)_{\Theta(2,3,9)},$
 $(20, 16, 22, 23, 47, 10, 35, 7, 2, 9, 31, 8, 32)_{\Theta(2,3,9)},$
 $(36, 28, 21, 9, 10, 14, 45, 16, 43, 25, 27, 47, 2)_{\Theta(2,3,9)},$
 $(16, 25, 23, 9, 51, 41, 20, 54, 3, 53, 19, 44, 15)_{\Theta(2,3,9)},$
 $(26, 18, 39, 15, 28, 0, 9, 54, 11, 2, 44, 8, 50)_{\Theta(2,3,9)},$
 $(13, 43, 7, 26, 11, 50, 10, 54, 0, 32, 15, 36, 3)_{\Theta(2,3,9)},$
 $(37, 23, 2, 25, 24, 21, 51, 22, 46, 27, 41, 18, 9)_{\Theta(2,3,9)},$
 $(1, 31, 26, 48, 25, 24, 39, 4, 19, 18, 52, 15, 20)_{\Theta(2,3,9)},$
 $(42, 26, 12, 21, 54, 4, 51, 14, 11, 1, 39, 23, 34)_{\Theta(2,3,9)},$
 $(0, 53, 17, 37, 24, 41, 6, 44, 25, 39, 5, 31, 27)_{\Theta(2,5,7)},$
 $(0, 19, 9, 23, 32, 5, 48, 38, 21, 26, 52, 18, 29)_{\Theta(2,5,7)},$
 $(47, 18, 9, 8, 41, 27, 40, 16, 26, 46, 0, 32, 1)_{\Theta(2,5,7)},$

$(32, 20, 21, 25, 34, 4, 39, 22, 16, 52, 12, 9, 6)_{\Theta(2,5,7)},$
 $(27, 7, 14, 45, 1, 53, 10, 47, 18, 43, 13, 35, 17)_{\Theta(2,5,7)},$
 $(31, 39, 16, 15, 36, 19, 2, 23, 35, 18, 17, 46, 12)_{\Theta(2,5,7)},$
 $(46, 14, 23, 18, 25, 3, 24, 21, 19, 50, 7, 54, 15)_{\Theta(2,5,7)},$
 $(29, 26, 4, 17, 24, 7, 42, 27, 54, 12, 34, 1, 50)_{\Theta(2,5,7)},$
 $(54, 43, 22, 13, 36, 12, 10, 0, 5, 33, 1, 14, 19)_{\Theta(2,5,7)},$
 $(24, 25, 45, 52, 19, 20, 11, 26, 53, 2, 43, 15, 12)_{\Theta(2,5,7)},$
 $(0, 40, 15, 48, 6, 46, 10, 29, 23, 1, 3, 53, 22)_{\Theta(2,5,7)},$
 $(0, 36, 15, 17, 40, 14, 26, 7, 21, 6, 41, 1, 8)_{\Theta(3,3,8)},$
 $(31, 35, 12, 2, 9, 15, 19, 32, 18, 33, 5, 46, 1)_{\Theta(3,3,8)},$
 $(29, 11, 6, 40, 26, 21, 16, 42, 7, 14, 33, 27, 4)_{\Theta(3,3,8)},$
 $(48, 1, 21, 30, 10, 47, 4, 50, 3, 2, 20, 31, 18)_{\Theta(3,3,8)},$
 $(34, 5, 3, 35, 26, 38, 23, 49, 10, 39, 13, 29, 15)_{\Theta(3,3,8)},$
 $(37, 25, 17, 8, 20, 49, 19, 6, 0, 5, 28, 4, 18)_{\Theta(3,3,8)},$
 $(39, 18, 15, 38, 14, 46, 24, 27, 16, 1, 33, 0, 34)_{\Theta(3,3,8)},$
 $(29, 23, 27, 32, 8, 44, 2, 51, 6, 25, 22, 52, 4)_{\Theta(3,3,8)},$
 $(33, 13, 25, 28, 23, 48, 11, 47, 12, 44, 27, 10, 7)_{\Theta(3,3,8)},$
 $(38, 21, 16, 46, 24, 23, 17, 20, 29, 4, 42, 15, 31)_{\Theta(3,3,8)},$
 $(27, 47, 22, 20, 55, 24, 42, 18, 36, 25, 26, 4, 5)_{\Theta(3,3,8)},$
 $(0, 23, 50, 2, 3, 18, 47, 28, 21, 34, 7, 32, 17)_{\Theta(3,4,7)},$
 $(20, 37, 30, 14, 39, 13, 11, 29, 16, 40, 10, 35, 5)_{\Theta(3,4,7)},$
 $(42, 10, 25, 48, 27, 32, 19, 13, 24, 2, 21, 52, 15)_{\Theta(3,4,7)},$
 $(7, 9, 28, 22, 12, 3, 47, 52, 5, 45, 6, 30, 24)_{\Theta(3,4,7)},$
 $(48, 37, 12, 23, 9, 14, 16, 21, 24, 10, 0, 51, 3)_{\Theta(3,4,7)},$
 $(6, 0, 47, 7, 38, 1, 53, 39, 4, 37, 21, 49, 2)_{\Theta(3,4,7)},$
 $(0, 10, 21, 7, 30, 25, 45, 40, 24, 44, 26, 48, 16)_{\Theta(3,4,7)},$
 $(50, 34, 3, 16, 15, 33, 11, 19, 47, 26, 1, 24, 23)_{\Theta(3,4,7)},$
 $(12, 8, 41, 26, 39, 25, 50, 51, 19, 2, 47, 13, 34)_{\Theta(3,4,7)},$
 $(42, 12, 2, 13, 6, 43, 21, 14, 45, 18, 25, 27, 29)_{\Theta(3,4,7)},$
 $(32, 27, 18, 17, 3, 41, 5, 6, 7, 43, 12, 55, 9)_{\Theta(3,4,7)},$
 $(0, 52, 18, 27, 37, 14, 30, 12, 25, 4, 10, 1, 3)_{\Theta(3,5,6)},$
 $(18, 4, 3, 36, 13, 30, 16, 33, 37, 1, 47, 6, 5)_{\Theta(3,5,6)},$
 $(10, 36, 39, 5, 7, 47, 19, 2, 3, 33, 17, 29, 9)_{\Theta(3,5,6)},$
 $(9, 19, 39, 14, 41, 8, 21, 30, 34, 10, 36, 17, 45)_{\Theta(3,5,6)},$
 $(7, 0, 34, 5, 41, 19, 42, 2, 45, 27, 21, 3, 48)_{\Theta(3,5,6)},$
 $(29, 41, 26, 27, 5, 31, 4, 14, 8, 43, 12, 38, 16)_{\Theta(3,5,6)},$
 $(51, 32, 3, 8, 27, 46, 10, 17, 14, 42, 12, 6, 9)_{\Theta(3,5,6)},$
 $(5, 54, 46, 11, 18, 39, 24, 23, 43, 21, 15, 17, 6)_{\Theta(3,5,6)},$
 $(38, 3, 0, 47, 5, 16, 23, 12, 6, 49, 8, 52, 13)_{\Theta(3,5,6)},$
 $(13, 43, 48, 20, 4, 2, 44, 7, 27, 24, 11, 40, 10)_{\Theta(3,5,6)},$
 $(2, 28, 41, 6, 47, 5, 54, 20, 16, 55, 23, 36, 18)_{\Theta(3,5,6)},$
 $(0, 54, 32, 9, 6, 36, 21, 10, 5, 19, 34, 25, 16)_{\Theta(4,5,5)},$
 $(9, 12, 51, 26, 49, 41, 24, 36, 19, 2, 23, 1, 31)_{\Theta(4,5,5)},$
 $(25, 33, 41, 26, 20, 47, 10, 43, 8, 44, 9, 40, 6)_{\Theta(4,5,5)},$
 $(14, 0, 46, 21, 29, 13, 23, 31, 27, 53, 6, 12, 39)_{\Theta(4,5,5)},$
 $(17, 28, 16, 30, 12, 26, 36, 23, 14, 45, 19, 44, 7)_{\Theta(4,5,5)},$
 $(53, 33, 22, 27, 21, 7, 35, 9, 11, 5, 44, 26, 0)_{\Theta(4,5,5)},$

$(26, 18, 8, 25, 54, 47, 7, 30, 3, 16, 38, 5, 19)_{\Theta(4,5,5)},$
 $(29, 36, 8, 42, 7, 15, 17, 30, 14, 19, 35, 1, 16)_{\Theta(4,5,5)},$
 $(7, 0, 24, 47, 23, 17, 20, 48, 15, 41, 11, 31, 2)_{\Theta(4,5,5)},$
 $(6, 38, 46, 15, 12, 41, 17, 44, 14, 32, 8, 22, 19)_{\Theta(4,5,5)},$
 $(0, 10, 5, 12, 55, 30, 2, 43, 25, 31, 21, 38, 27)_{\Theta(4,5,5)}$

under the action of the mapping $x \mapsto x + 4 \pmod{28}$ for $x < 28$, $x \mapsto (x + 4 \pmod{28}) + 28$ for $x \geq 28$. \square

F Theta graphs of 15 edges

Proof of Lemma 3.11

K_{15} Let the vertex set be $Z_{14} \cup \{\infty\}$. The decompositions consist of the graphs

$(0, 2, 1, 3, 5, 8, 4, 9, \infty, 12, 6, 13, 7, 11)_{\Theta(1,2,12)},$
 $(0, 2, 1, 5, 4, 3, 6, 12, 7, 13, 8, \infty, 11, 9)_{\Theta(1,3,11)},$
 $(0, 2, 1, 3, 6, 5, 9, 10, 13, 7, 12, 4, 11, \infty)_{\Theta(1,4,10)},$
 $(0, 2, 1, 3, 4, 7, 6, 10, \infty, 13, 9, 12, 5, 11)_{\Theta(1,5,9)},$
 $(0, 2, 1, 3, 4, 7, 10, 9, 5, 11, 6, 13, \infty, 12)_{\Theta(1,6,8)},$
 $(0, 2, 1, 3, 4, 7, 11, 6, 9, 12, 5, 13, \infty, 8)_{\Theta(1,7,7)},$
 $(0, 2, 1, 4, 3, 5, 8, \infty, 11, 6, 12, 7, 13, 9)_{\Theta(2,2,11)},$
 $(0, 2, 1, 3, 6, 5, 7, 13, 9, \infty, 8, 10, 4, 11)_{\Theta(2,3,10)},$
 $(0, 2, 1, 3, 5, 8, 4, 6, 11, \infty, 12, 7, 13, 9)_{\Theta(2,4,9)},$
 $(0, 2, 1, 3, 5, 8, 4, 6, 11, \infty, 12, 7, 13, 9)_{\Theta(2,5,8)},$
 $(0, 2, 1, 3, 5, 8, 4, 9, 12, 6, \infty, 13, 7, 11)_{\Theta(2,6,7)},$
 $(0, 2, 1, 4, 3, 7, 6, 10, 5, 13, 11, 12, \infty, 9)_{\Theta(3,3,9)},$
 $(0, 2, 1, 4, 3, 5, 9, 6, \infty, 7, 13, 8, 12, 11)_{\Theta(3,4,8)},$
 $(0, 2, 1, 4, 3, 5, 6, 11, 7, 13, 9, \infty, 12, 8)_{\Theta(3,5,7)},$
 $(0, 2, 1, 4, 3, 5, 6, 11, \infty, 8, 12, 7, 13, 9)_{\Theta(3,6,6)},$
 $(0, 2, 1, 3, 4, 5, 8, 11, 6, 10, \infty, 7, 13, 9)_{\Theta(4,4,7)},$
 $(0, 2, 1, 3, 4, 5, 8, \infty, 9, 6, 10, 13, 7, 11)_{\Theta(4,5,6)},$
 $(0, 2, 1, 3, 4, 6, 5, 10, \infty, 9, 8, 11, 7, 13)_{\Theta(5,5,5)}$

under the action of the mapping $x \mapsto x + 2 \pmod{14}$, $\infty \mapsto \infty$.

K_{16} Let the vertex set be Z_{16} . The decompositions consist of the graphs

$(0, 1, 15, 8, 2, 9, 12, 7, 3, 4, 10, 5, 14, 11)_{\Theta(1,2,12)},$
 $(2, 3, 14, 6, 9, 11, 4, 7, 13, 8, 5, 15, 12, 1)_{\Theta(1,2,12)},$
 $(4, 5, 0, 13, 11, 8, 6, 1, 14, 9, 3, 15, 10, 12)_{\Theta(1,2,12)},$
 $(6, 7, 5, 15, 8, 4, 12, 0, 13, 3, 10, 9, 1, 2)_{\Theta(1,2,12)},$
 $(8, 9, 7, 10, 6, 14, 4, 1, 5, 11, 0, 2, 13, 15)_{\Theta(1,2,12)},$
 $(10, 11, 7, 14, 0, 9, 5, 3, 8, 1, 13, 6, 12, 2)_{\Theta(1,2,12)},$
 $(12, 13, 14, 3, 0, 7, 1, 10, 2, 15, 11, 6, 4, 9)_{\Theta(1,2,12)},$
 $(14, 15, 7, 8, 12, 11, 3, 6, 0, 10, 13, 5, 2, 4)_{\Theta(1,2,12)},$
 $(0, 1, 15, 11, 13, 5, 3, 7, 14, 8, 10, 2, 12, 6)_{\Theta(1,3,11)},$

$(2, 3, 8, 12, 7, 1, 4, 14, 10, 0, 6, 13, 9, 15)_{\Theta(1,3,11)},$
 $(4, 5, 7, 9, 11, 2, 1, 3, 0, 12, 10, 6, 8, 15)_{\Theta(1,3,11)},$
 $(6, 7, 11, 0, 5, 1, 9, 4, 3, 10, 15, 12, 14, 13)_{\Theta(1,3,11)},$
 $(8, 9, 4, 6, 0, 5, 11, 14, 1, 13, 10, 7, 15, 2)_{\Theta(1,3,11)},$
 $(10, 11, 4, 13, 5, 2, 0, 14, 6, 3, 9, 12, 1, 8)_{\Theta(1,3,11)},$
 $(12, 13, 4, 2, 7, 11, 3, 8, 5, 14, 9, 10, 1, 15)_{\Theta(1,3,11)},$
 $(14, 15, 2, 6, 3, 13, 8, 7, 5, 12, 11, 9, 0, 4)_{\Theta(1,3,11)},$
 $(0, 1, 9, 12, 11, 5, 7, 10, 14, 8, 6, 4, 3, 15)_{\Theta(1,4,10)},$
 $(2, 3, 11, 6, 9, 15, 13, 0, 7, 14, 12, 1, 4, 8)_{\Theta(1,4,10)},$
 $(4, 5, 2, 10, 8, 12, 6, 13, 9, 14, 1, 3, 0, 11)_{\Theta(1,4,10)},$
 $(6, 7, 5, 12, 8, 3, 13, 11, 4, 15, 0, 10, 9, 2)_{\Theta(1,4,10)},$
 $(8, 9, 11, 14, 4, 1, 7, 13, 2, 6, 10, 12, 15, 5)_{\Theta(1,4,10)},$
 $(10, 11, 13, 1, 9, 4, 0, 2, 5, 3, 14, 6, 15, 7)_{\Theta(1,4,10)},$
 $(12, 13, 3, 10, 5, 7, 9, 15, 8, 0, 6, 1, 2, 14)_{\Theta(1,4,10)},$
 $(14, 15, 5, 1, 10, 0, 12, 2, 8, 13, 4, 7, 3, 11)_{\Theta(1,4,10)},$
 $(0, 1, 15, 13, 4, 7, 12, 2, 11, 6, 5, 8, 14, 10)_{\Theta(1,5,9)},$
 $(2, 3, 14, 5, 11, 9, 15, 12, 1, 6, 4, 10, 0, 7)_{\Theta(1,5,9)},$
 $(4, 5, 11, 8, 2, 13, 15, 10, 7, 9, 6, 14, 1, 3)_{\Theta(1,5,9)},$
 $(6, 7, 8, 10, 9, 13, 15, 3, 11, 12, 14, 0, 2, 5)_{\Theta(1,5,9)},$
 $(8, 9, 13, 10, 3, 4, 0, 11, 1, 15, 5, 12, 6, 2)_{\Theta(1,5,9)},$
 $(10, 11, 5, 1, 9, 14, 6, 13, 0, 3, 8, 4, 12, 7)_{\Theta(1,5,9)},$
 $(12, 13, 9, 0, 6, 3, 10, 2, 4, 1, 8, 15, 7, 14)_{\Theta(1,5,9)},$
 $(14, 15, 4, 0, 5, 9, 3, 12, 8, 7, 2, 1, 13, 11)_{\Theta(1,5,9)},$
 $(0, 1, 7, 13, 11, 4, 10, 14, 9, 2, 15, 5, 3, 6)_{\Theta(1,6,8)},$
 $(2, 3, 12, 6, 10, 13, 14, 8, 7, 4, 0, 5, 11, 9)_{\Theta(1,6,8)},$
 $(4, 5, 1, 13, 6, 15, 12, 14, 10, 8, 0, 11, 3, 7)_{\Theta(1,6,8)},$
 $(6, 7, 5, 1, 2, 10, 12, 0, 13, 9, 4, 15, 8, 11)_{\Theta(1,6,8)},$
 $(8, 9, 1, 14, 11, 2, 7, 13, 4, 12, 0, 3, 15, 10)_{\Theta(1,6,8)},$
 $(10, 11, 0, 9, 1, 3, 12, 5, 8, 4, 6, 2, 14, 15)_{\Theta(1,6,8)},$
 $(12, 14, 9, 15, 0, 2, 5, 13, 3, 8, 6, 11, 1, 7)_{\Theta(1,6,8)},$
 $(13, 15, 2, 4, 3, 10, 7, 5, 9, 6, 14, 8, 12, 1)_{\Theta(1,6,8)},$
 $(0, 1, 3, 6, 8, 4, 12, 5, 14, 7, 10, 9, 2, 15)_{\Theta(1,7,7)},$
 $(2, 3, 1, 8, 13, 0, 12, 11, 4, 15, 10, 5, 6, 9)_{\Theta(1,7,7)},$
 $(4, 5, 1, 9, 15, 12, 13, 11, 14, 2, 6, 10, 3, 7)_{\Theta(1,7,7)},$
 $(6, 7, 11, 0, 10, 12, 8, 15, 13, 3, 1, 14, 9, 4)_{\Theta(1,7,7)},$
 $(8, 9, 14, 13, 10, 1, 7, 0, 11, 2, 12, 3, 15, 5)_{\Theta(1,7,7)},$
 $(10, 11, 2, 13, 7, 8, 3, 4, 14, 5, 0, 6, 12, 9)_{\Theta(1,7,7)},$
 $(12, 14, 1, 13, 4, 6, 15, 11, 7, 2, 0, 8, 5, 3)_{\Theta(1,7,7)},$
 $(13, 15, 5, 2, 8, 10, 4, 0, 9, 7, 11, 1, 6, 14)_{\Theta(1,7,7)},$
 $(0, 1, 7, 4, 12, 10, 5, 6, 14, 13, 8, 3, 9, 11)_{\Theta(2,2,11)},$
 $(2, 3, 7, 10, 0, 8, 12, 4, 5, 13, 1, 14, 9, 15)_{\Theta(2,2,11)},$
 $(4, 5, 14, 8, 15, 2, 12, 13, 11, 7, 6, 0, 3, 1)_{\Theta(2,2,11)},$
 $(6, 7, 8, 9, 10, 15, 1, 2, 3, 11, 4, 13, 0, 5)_{\Theta(2,2,11)},$
 $(8, 9, 1, 2, 11, 6, 4, 10, 7, 15, 12, 14, 3, 5)_{\Theta(2,2,11)},$
 $(10, 11, 14, 0, 1, 6, 2, 5, 12, 7, 4, 9, 13, 15)_{\Theta(2,2,11)},$
 $(12, 13, 3, 6, 1, 0, 9, 8, 10, 11, 5, 15, 14, 2)_{\Theta(2,2,11)},$

$(14, 15, 0, 8, 7, 13, 10, 9, 12, 11, 2, 4, 3, 6)_{\Theta(2,2,11)},$
 $(0, 1, 6, 10, 5, 7, 4, 13, 8, 12, 15, 3, 11, 14)_{\Theta(2,3,10)},$
 $(2, 3, 10, 14, 6, 15, 4, 8, 9, 11, 7, 1, 0, 13)_{\Theta(2,3,10)},$
 $(4, 5, 9, 10, 13, 3, 1, 8, 15, 11, 6, 7, 12, 2)_{\Theta(2,3,10)},$
 $(6, 7, 2, 13, 15, 5, 14, 12, 4, 11, 10, 9, 0, 3)_{\Theta(2,3,10)},$
 $(8, 9, 2, 5, 7, 10, 6, 4, 1, 13, 14, 15, 0, 12)_{\Theta(2,3,10)},$
 $(10, 11, 12, 14, 0, 7, 8, 6, 9, 3, 5, 4, 2, 1)_{\Theta(2,3,10)},$
 $(12, 15, 6, 1, 10, 13, 2, 11, 5, 0, 8, 3, 14, 9)_{\Theta(2,3,10)},$
 $(13, 14, 7, 11, 8, 9, 1, 15, 5, 12, 3, 2, 0, 4)_{\Theta(2,3,10)},$
 $(0, 1, 3, 14, 4, 9, 6, 15, 13, 7, 5, 2, 10, 8)_{\Theta(2,4,9)},$
 $(2, 3, 11, 12, 5, 8, 14, 9, 0, 15, 4, 13, 1, 7)_{\Theta(2,4,9)},$
 $(4, 5, 11, 0, 2, 1, 3, 14, 12, 9, 6, 8, 13, 10)_{\Theta(2,4,9)},$
 $(6, 7, 10, 4, 5, 9, 3, 2, 13, 11, 8, 12, 1, 15)_{\Theta(2,4,9)},$
 $(8, 9, 15, 4, 10, 11, 14, 1, 6, 2, 7, 0, 12, 3)_{\Theta(2,4,9)},$
 $(10, 11, 1, 0, 13, 12, 3, 5, 14, 6, 7, 8, 2, 15)_{\Theta(2,4,9)},$
 $(12, 14, 10, 4, 7, 11, 6, 5, 0, 8, 9, 13, 3, 15)_{\Theta(2,4,9)},$
 $(13, 15, 5, 14, 7, 12, 6, 11, 0, 1, 4, 2, 9, 10)_{\Theta(2,4,9)},$
 $(0, 1, 6, 5, 8, 12, 13, 9, 15, 2, 11, 4, 10, 3)_{\Theta(2,5,8)},$
 $(2, 3, 8, 0, 1, 7, 6, 5, 4, 9, 11, 14, 12, 15)_{\Theta(2,5,8)},$
 $(4, 5, 6, 0, 8, 1, 10, 7, 14, 3, 11, 13, 9, 12)_{\Theta(2,5,8)},$
 $(6, 7, 10, 12, 4, 1, 9, 8, 11, 15, 0, 13, 14, 2)_{\Theta(2,5,8)},$
 $(8, 9, 10, 15, 5, 1, 2, 7, 11, 6, 14, 4, 13, 3)_{\Theta(2,5,8)},$
 $(10, 11, 12, 0, 3, 7, 5, 13, 2, 4, 8, 14, 15, 1)_{\Theta(2,5,8)},$
 $(12, 13, 7, 3, 4, 15, 6, 2, 10, 11, 0, 14, 9, 5)_{\Theta(2,5,8)},$
 $(14, 15, 10, 1, 12, 0, 7, 5, 3, 2, 6, 9, 8, 13)_{\Theta(2,5,8)},$
 $(0, 1, 11, 6, 9, 5, 7, 10, 14, 4, 12, 8, 13, 2)_{\Theta(2,6,7)},$
 $(2, 3, 6, 15, 7, 8, 1, 13, 5, 12, 9, 11, 10, 14)_{\Theta(2,6,7)},$
 $(4, 5, 3, 6, 11, 14, 8, 15, 7, 9, 10, 0, 12, 13)_{\Theta(2,6,7)},$
 $(6, 7, 1, 5, 14, 15, 13, 0, 8, 9, 4, 2, 10, 3)_{\Theta(2,6,7)},$
 $(8, 9, 0, 10, 13, 7, 12, 1, 11, 5, 4, 15, 3, 2)_{\Theta(2,6,7)},$
 $(10, 11, 4, 5, 8, 2, 0, 3, 6, 7, 14, 1, 15, 12)_{\Theta(2,6,7)},$
 $(12, 13, 6, 3, 1, 5, 0, 4, 14, 2, 7, 11, 15, 9)_{\Theta(2,6,7)},$
 $(14, 15, 6, 13, 11, 2, 12, 10, 9, 3, 8, 4, 1, 0)_{\Theta(2,6,7)},$
 $(0, 1, 10, 2, 8, 9, 6, 15, 7, 11, 12, 3, 4, 5)_{\Theta(3,3,9)},$
 $(2, 3, 12, 15, 4, 1, 14, 13, 10, 7, 8, 11, 5, 0)_{\Theta(3,3,9)},$
 $(4, 5, 8, 3, 0, 13, 9, 10, 14, 6, 7, 1, 15, 2)_{\Theta(3,3,9)},$
 $(6, 7, 11, 4, 5, 14, 10, 8, 15, 0, 2, 13, 12, 9)_{\Theta(3,3,9)},$
 $(8, 9, 13, 3, 6, 2, 5, 12, 14, 0, 1, 10, 15, 11)_{\Theta(3,3,9)},$
 $(10, 11, 3, 14, 4, 13, 12, 8, 2, 7, 5, 9, 6, 1)_{\Theta(3,3,9)},$
 $(12, 15, 1, 14, 6, 4, 0, 9, 13, 7, 3, 11, 10, 5)_{\Theta(3,3,9)},$
 $(13, 14, 1, 8, 15, 9, 6, 3, 2, 11, 0, 7, 12, 4)_{\Theta(3,3,9)},$
 $(0, 1, 2, 15, 7, 12, 11, 10, 14, 13, 5, 8, 6, 4)_{\Theta(3,4,8)},$
 $(2, 3, 9, 10, 11, 6, 1, 13, 7, 4, 15, 12, 8, 0)_{\Theta(3,4,8)},$
 $(4, 5, 13, 9, 2, 14, 12, 8, 3, 15, 7, 10, 11, 0)_{\Theta(3,4,8)},$
 $(6, 7, 0, 1, 5, 14, 8, 15, 9, 3, 11, 4, 10, 2)_{\Theta(3,4,8)},$
 $(8, 9, 10, 6, 1, 12, 0, 13, 15, 11, 5, 2, 3, 14)_{\Theta(3,4,8)},$

$(10, 11, 5, 7, 12, 9, 8, 1, 13, 6, 3, 4, 0, 14)_{\Theta(3,4,8)},$
 $(12, 15, 4, 5, 3, 13, 0, 6, 14, 7, 9, 1, 2, 8)_{\Theta(3,4,8)},$
 $(13, 14, 10, 15, 11, 9, 4, 12, 2, 6, 7, 3, 5, 1)_{\Theta(3,4,8)},$
 $(0, 1, 14, 7, 6, 8, 15, 9, 11, 2, 4, 12, 13, 5)_{\Theta(3,5,7)},$
 $(2, 3, 10, 12, 1, 0, 7, 13, 9, 8, 4, 6, 11, 15)_{\Theta(3,5,7)},$
 $(4, 5, 7, 6, 1, 8, 10, 14, 11, 3, 0, 2, 13, 15)_{\Theta(3,5,7)},$
 $(6, 7, 10, 9, 1, 12, 0, 15, 2, 14, 8, 11, 5, 3)_{\Theta(3,5,7)},$
 $(8, 9, 13, 14, 5, 0, 10, 11, 3, 4, 15, 2, 12, 6)_{\Theta(3,5,7)},$
 $(10, 11, 5, 12, 3, 9, 13, 1, 7, 2, 8, 0, 4, 14)_{\Theta(3,5,7)},$
 $(12, 15, 14, 6, 9, 4, 13, 10, 8, 7, 5, 2, 3, 1)_{\Theta(3,5,7)},$
 $(13, 14, 6, 3, 11, 7, 12, 15, 0, 9, 5, 4, 10, 1)_{\Theta(3,5,7)},$
 $(0, 1, 2, 3, 15, 6, 12, 11, 7, 8, 14, 4, 13, 5)_{\Theta(3,6,6)},$
 $(2, 3, 4, 7, 11, 8, 9, 12, 14, 6, 10, 0, 1, 13)_{\Theta(3,6,6)},$
 $(4, 5, 3, 15, 1, 12, 10, 9, 2, 0, 6, 7, 13, 8)_{\Theta(3,6,6)},$
 $(6, 7, 13, 15, 9, 1, 2, 14, 10, 8, 12, 4, 11, 5)_{\Theta(3,6,6)},$
 $(8, 9, 15, 4, 7, 2, 10, 11, 0, 3, 5, 6, 1, 14)_{\Theta(3,6,6)},$
 $(10, 11, 15, 14, 1, 8, 4, 5, 9, 13, 12, 0, 3, 6)_{\Theta(3,6,6)},$
 $(12, 13, 7, 14, 2, 8, 10, 5, 0, 3, 9, 15, 1, 11)_{\Theta(3,6,6)},$
 $(14, 15, 5, 12, 0, 7, 9, 13, 2, 6, 4, 10, 3, 11)_{\Theta(3,6,6)},$
 $(0, 1, 12, 10, 14, 2, 7, 15, 4, 8, 9, 5, 11, 6)_{\Theta(4,4,7)},$
 $(2, 3, 8, 5, 15, 12, 1, 11, 13, 4, 10, 7, 14, 0)_{\Theta(4,4,7)},$
 $(4, 5, 3, 1, 7, 2, 14, 13, 12, 6, 15, 0, 9, 10)_{\Theta(4,4,7)},$
 $(6, 7, 9, 2, 11, 3, 12, 8, 13, 0, 1, 10, 15, 4)_{\Theta(4,4,7)},$
 $(8, 9, 11, 4, 14, 10, 13, 15, 1, 5, 6, 2, 3, 7)_{\Theta(4,4,7)},$
 $(10, 11, 2, 1, 9, 6, 7, 13, 0, 5, 3, 8, 14, 12)_{\Theta(4,4,7)},$
 $(12, 14, 7, 0, 6, 9, 4, 5, 15, 8, 13, 3, 10, 11)_{\Theta(4,4,7)},$
 $(13, 15, 9, 3, 14, 12, 5, 2, 1, 4, 6, 8, 0, 11)_{\Theta(4,4,7)},$
 $(0, 1, 2, 11, 6, 13, 10, 7, 3, 5, 15, 4, 14, 12)_{\Theta(4,5,6)},$
 $(2, 3, 15, 14, 13, 9, 12, 8, 11, 5, 10, 1, 0, 6)_{\Theta(4,5,6)},$
 $(4, 5, 7, 1, 8, 13, 11, 10, 14, 6, 12, 0, 3, 9)_{\Theta(4,5,6)},$
 $(6, 7, 5, 3, 15, 14, 0, 9, 8, 10, 4, 12, 13, 2)_{\Theta(4,5,6)},$
 $(8, 9, 13, 5, 1, 14, 7, 0, 4, 10, 2, 12, 11, 15)_{\Theta(4,5,6)},$
 $(10, 11, 3, 14, 1, 9, 7, 5, 4, 15, 6, 2, 8, 0)_{\Theta(4,5,6)},$
 $(12, 13, 7, 11, 9, 3, 4, 2, 1, 10, 0, 15, 8, 6)_{\Theta(4,5,6)},$
 $(14, 15, 11, 5, 12, 9, 6, 7, 13, 2, 3, 8, 4, 1)_{\Theta(4,5,6)},$
 $(0, 1, 10, 9, 6, 2, 7, 12, 14, 11, 15, 4, 5, 8)_{\Theta(5,5,5)},$
 $(2, 3, 15, 13, 8, 7, 11, 6, 1, 12, 10, 4, 9, 5)_{\Theta(5,5,5)},$
 $(4, 5, 1, 7, 9, 14, 8, 10, 15, 12, 3, 0, 2, 13)_{\Theta(5,5,5)},$
 $(6, 7, 15, 8, 3, 10, 0, 14, 2, 4, 12, 9, 11, 13)_{\Theta(5,5,5)},$
 $(8, 9, 14, 15, 11, 3, 6, 13, 4, 0, 2, 7, 5, 1)_{\Theta(5,5,5)},$
 $(10, 11, 13, 0, 8, 12, 6, 7, 14, 4, 1, 3, 2, 5)_{\Theta(5,5,5)},$
 $(12, 13, 0, 5, 15, 1, 4, 6, 14, 3, 10, 11, 8, 9)_{\Theta(5,5,5)},$
 $(14, 15, 1, 0, 11, 7, 10, 5, 6, 3, 13, 12, 2, 9)_{\Theta(5,5,5)}.$

K_{21} Let the vertex set be Z_{21} . The decompositions consist of the graphs

$(0, 9, 7, 13, 18, 19, 5, 10, 2, 1, 11, 12, 15, 17)_{\Theta(1,2,12)},$
 $(5, 8, 15, 9, 3, 7, 16, 6, 11, 17, 13, 19, 1, 20)_{\Theta(1,2,12)},$

$(0, 14, 4, 18, 9, 10, 16, 6, 19, 11, 13, 1, 8, 5)_{\Theta(1,3,11)},$
 $(12, 11, 14, 6, 15, 9, 7, 2, 1, 19, 8, 18, 13, 17)_{\Theta(1,3,11)},$
 $(0, 11, 8, 6, 16, 5, 2, 13, 12, 15, 14, 1, 18, 10)_{\Theta(1,4,10)},$
 $(0, 14, 9, 5, 7, 15, 1, 3, 19, 4, 13, 16, 20, 8)_{\Theta(1,4,10)},$
 $(0, 1, 2, 10, 8, 19, 17, 13, 18, 3, 15, 5, 11, 12)_{\Theta(1,5,9)},$
 $(0, 7, 4, 12, 17, 14, 19, 10, 11, 3, 6, 20, 8, 13)_{\Theta(1,5,9)},$
 $(0, 18, 11, 14, 10, 8, 19, 5, 3, 7, 12, 4, 17, 9)_{\Theta(1,6,8)},$
 $(11, 5, 2, 7, 16, 13, 6, 10, 4, 15, 8, 1, 3, 9)_{\Theta(1,6,8)},$
 $(0, 3, 10, 15, 9, 5, 12, 16, 2, 13, 14, 19, 11, 4)_{\Theta(1,7,7)},$
 $(2, 11, 17, 18, 6, 4, 1, 13, 5, 15, 20, 16, 10, 3)_{\Theta(1,7,7)},$
 $(0, 11, 7, 1, 3, 2, 18, 8, 5, 12, 10, 17, 15, 19)_{\Theta(2,2,11)},$
 $(5, 12, 18, 4, 17, 19, 16, 6, 15, 10, 1, 7, 2, 8)_{\Theta(2,2,11)},$
 $(0, 11, 17, 14, 4, 6, 8, 5, 1, 9, 10, 16, 12, 19)_{\Theta(2,3,10)},$
 $(17, 15, 6, 1, 13, 18, 2, 14, 16, 19, 20, 12, 9, 4)_{\Theta(2,3,10)},$
 $(0, 17, 13, 5, 7, 3, 10, 15, 16, 2, 1, 19, 12, 8)_{\Theta(2,4,9)},$
 $(5, 9, 18, 2, 17, 6, 3, 1, 7, 16, 8, 19, 14, 15)_{\Theta(2,4,9)},$
 $(0, 6, 15, 2, 7, 17, 4, 3, 10, 8, 11, 18, 1, 5)_{\Theta(2,5,8)},$
 $(12, 15, 2, 13, 19, 16, 4, 7, 8, 1, 9, 5, 11, 20)_{\Theta(2,5,8)},$
 $(0, 9, 1, 12, 14, 17, 2, 13, 7, 19, 5, 15, 8, 3)_{\Theta(2,6,7)},$
 $(17, 11, 16, 18, 14, 5, 13, 3, 19, 4, 1, 6, 9, 7)_{\Theta(2,6,7)},$
 $(0, 19, 4, 8, 15, 7, 3, 17, 10, 11, 5, 14, 18, 13)_{\Theta(3,3,9)},$
 $(7, 9, 4, 18, 6, 17, 5, 0, 2, 10, 12, 11, 14, 19)_{\Theta(3,3,9)},$
 $(0, 4, 18, 7, 19, 13, 11, 8, 2, 3, 9, 16, 17, 14)_{\Theta(3,4,8)},$
 $(15, 16, 6, 7, 11, 2, 0, 8, 18, 10, 5, 13, 17, 12)_{\Theta(3,4,8)},$
 $(0, 4, 6, 1, 2, 10, 9, 8, 3, 17, 12, 19, 13, 11)_{\Theta(3,5,7)},$
 $(12, 4, 3, 14, 8, 0, 13, 15, 16, 7, 2, 20, 11, 5)_{\Theta(3,5,7)},$
 $(0, 3, 7, 5, 10, 19, 18, 12, 8, 16, 11, 1, 14, 17)_{\Theta(3,6,6)},$
 $(17, 8, 11, 4, 5, 15, 12, 3, 7, 18, 10, 13, 19, 0)_{\Theta(3,6,6)},$
 $(0, 18, 19, 12, 8, 16, 15, 2, 6, 3, 13, 10, 17, 11)_{\Theta(4,4,7)},$
 $(7, 10, 3, 12, 4, 2, 11, 1, 5, 6, 8, 16, 17, 14)_{\Theta(4,4,7)},$
 $(0, 3, 1, 12, 11, 19, 7, 17, 8, 2, 16, 13, 9, 15)_{\Theta(4,5,6)},$
 $(7, 16, 8, 4, 9, 13, 0, 3, 14, 20, 6, 2, 5, 11)_{\Theta(4,5,6)},$
 $(0, 1, 5, 15, 12, 7, 4, 17, 14, 2, 8, 10, 13, 3)_{\Theta(5,5,5)},$
 $(19, 18, 8, 14, 15, 11, 12, 3, 9, 10, 2, 16, 4, 20)_{\Theta(5,5,5)}$

under the action of the mapping $x \mapsto x + 3 \pmod{21}$.

K_{25} Let the vertex set be Z_{25} . The decompositions consist of the graphs

$(0, 19, 3, 12, 7, 21, 1, 2, 6, 8, 11, 20, 9, 13)_{\Theta(1,2,12)},$
 $(4, 1, 11, 14, 9, 17, 15, 8, 3, 21, 12, 22, 23, 13)_{\Theta(1,2,12)},$
 $(10, 2, 14, 20, 21, 19, 22, 15, 11, 5, 18, 12, 23, 9)_{\Theta(1,2,12)},$
 $(21, 2, 4, 9, 8, 17, 20, 0, 11, 3, 5, 14, 15, 23)_{\Theta(1,2,12)},$
 $(0, 5, 10, 1, 9, 11, 14, 6, 16, 23, 17, 15, 21, 22)_{\Theta(1,3,11)},$
 $(8, 10, 15, 23, 5, 19, 6, 3, 13, 4, 11, 7, 2, 9)_{\Theta(1,3,11)},$

$(22, 15, 6, 1, 10, 14, 19, 13, 9, 7, 18, 4, 3, 16)_{\Theta(1,3,11)},$
 $(23, 2, 18, 16, 6, 12, 9, 22, 14, 4, 10, 7, 8, 17)_{\Theta(1,3,11)},$
 $(0, 4, 23, 16, 11, 12, 7, 21, 8, 3, 20, 6, 10, 19)_{\Theta(1,4,10)},$
 $(6, 3, 12, 8, 2, 15, 20, 9, 7, 0, 10, 4, 22, 1)_{\Theta(1,4,10)},$
 $(0, 6, 1, 4, 23, 3, 17, 8, 19, 2, 11, 21, 9, 14)_{\Theta(1,4,10)},$
 $(9, 18, 22, 20, 8, 13, 14, 16, 17, 7, 10, 2, 24, 0)_{\Theta(1,4,10)},$
 $(0, 5, 4, 22, 17, 1, 18, 23, 21, 14, 16, 13, 9, 7)_{\Theta(1,5,9)},$
 $(10, 2, 23, 16, 1, 15, 0, 9, 18, 17, 14, 20, 21, 13)_{\Theta(1,5,9)},$
 $(6, 0, 12, 15, 23, 7, 14, 9, 19, 13, 1, 22, 18, 3)_{\Theta(1,5,9)},$
 $(11, 22, 20, 9, 6, 7, 16, 4, 17, 23, 0, 24, 13, 14)_{\Theta(1,5,9)},$
 $(0, 1, 19, 13, 9, 20, 12, 16, 11, 15, 2, 22, 14, 4)_{\Theta(1,6,8)},$
 $(14, 1, 7, 0, 13, 18, 16, 12, 10, 21, 19, 20, 5, 2)_{\Theta(1,6,8)},$
 $(14, 13, 3, 21, 18, 12, 2, 6, 0, 9, 22, 23, 7, 4)_{\Theta(1,6,8)},$
 $(18, 0, 1, 17, 21, 2, 23, 3, 16, 9, 14, 10, 5, 8)_{\Theta(1,6,8)},$
 $(0, 12, 16, 22, 8, 2, 17, 20, 21, 13, 10, 9, 4, 3)_{\Theta(1,7,7)},$
 $(2, 4, 14, 8, 19, 10, 17, 16, 7, 21, 3, 15, 20, 13)_{\Theta(1,7,7)},$
 $(9, 16, 2, 3, 18, 10, 20, 14, 19, 5, 11, 23, 21, 1)_{\Theta(1,7,7)},$
 $(20, 6, 18, 23, 1, 17, 13, 9, 24, 2, 19, 11, 7, 5)_{\Theta(1,7,7)},$
 $(0, 15, 21, 14, 16, 2, 7, 3, 5, 22, 20, 8, 17, 10)_{\Theta(2,2,11)},$
 $(16, 4, 13, 22, 18, 0, 12, 23, 20, 17, 21, 6, 7, 19)_{\Theta(2,2,11)},$
 $(21, 13, 14, 3, 4, 20, 10, 11, 16, 8, 0, 19, 17, 9)_{\Theta(2,2,11)},$
 $(18, 4, 6, 23, 12, 9, 21, 10, 14, 19, 8, 7, 17, 1)_{\Theta(2,2,11)},$
 $(0, 6, 2, 3, 22, 7, 21, 4, 19, 5, 23, 1, 13, 8)_{\Theta(2,3,10)},$
 $(9, 11, 4, 13, 21, 8, 14, 1, 5, 18, 2, 22, 23, 0)_{\Theta(2,3,10)},$
 $(19, 20, 12, 2, 14, 17, 3, 18, 11, 5, 10, 22, 16, 21)_{\Theta(2,3,10)},$
 $(0, 10, 9, 4, 1, 15, 12, 2, 24, 8, 19, 21, 22, 18)_{\Theta(2,3,10)},$
 $(0, 19, 12, 9, 23, 6, 11, 17, 21, 1, 3, 22, 15, 14)_{\Theta(2,4,9)},$
 $(6, 7, 4, 13, 15, 5, 16, 9, 18, 8, 11, 10, 14, 2)_{\Theta(2,4,9)},$
 $(19, 10, 13, 4, 8, 15, 16, 3, 7, 23, 22, 14, 12, 1)_{\Theta(2,4,9)},$
 $(11, 17, 2, 15, 9, 20, 5, 18, 7, 6, 14, 13, 8, 0)_{\Theta(2,4,9)},$
 $(0, 9, 5, 22, 12, 16, 8, 19, 17, 4, 21, 11, 18, 13)_{\Theta(2,5,8)},$
 $(4, 18, 6, 19, 3, 12, 17, 1, 14, 9, 16, 0, 23, 10)_{\Theta(2,5,8)},$
 $(13, 23, 2, 19, 12, 9, 20, 7, 21, 1, 15, 14, 22, 5)_{\Theta(2,5,8)},$
 $(5, 8, 11, 7, 0, 9, 23, 1, 2, 21, 20, 10, 22, 6)_{\Theta(2,5,8)},$
 $(0, 23, 9, 16, 4, 22, 15, 10, 1, 3, 20, 19, 13, 8)_{\Theta(2,6,7)},$
 $(20, 8, 10, 13, 6, 9, 11, 12, 1, 22, 3, 17, 4, 21)_{\Theta(2,6,7)},$
 $(10, 12, 14, 7, 8, 4, 13, 22, 2, 11, 15, 1, 23, 6)_{\Theta(2,6,7)},$
 $(10, 17, 22, 12, 1, 16, 21, 14, 4, 15, 18, 19, 24, 9)_{\Theta(2,6,7)},$
 $(0, 6, 4, 9, 15, 7, 14, 1, 19, 10, 13, 12, 17, 11)_{\Theta(3,3,9)},$
 $(5, 23, 11, 8, 1, 18, 0, 22, 19, 7, 3, 17, 6, 16)_{\Theta(3,3,9)},$
 $(23, 6, 17, 8, 15, 14, 11, 10, 12, 2, 20, 18, 4, 22)_{\Theta(3,3,9)},$
 $(4, 19, 6, 2, 3, 9, 8, 15, 1, 10, 23, 14, 12, 0)_{\Theta(3,3,9)},$
 $(0, 17, 5, 15, 16, 7, 22, 8, 1, 21, 20, 9, 2, 13)_{\Theta(3,4,8)},$
 $(21, 20, 9, 14, 2, 4, 12, 15, 13, 1, 16, 18, 17, 23)_{\Theta(3,4,8)},$
 $(9, 21, 23, 18, 12, 3, 4, 19, 1, 2, 5, 17, 6, 10)_{\Theta(3,4,8)},$

$(7, 9, 19, 3, 11, 14, 5, 0, 18, 8, 4, 6, 23, 10)_{\Theta(3,4,8)},$
 $(0, 16, 3, 13, 2, 17, 1, 9, 12, 6, 4, 19, 5, 20)_{\Theta(3,5,7)},$
 $(20, 17, 18, 10, 12, 1, 21, 3, 4, 7, 8, 9, 13, 22)_{\Theta(3,5,7)},$
 $(2, 20, 23, 14, 1, 3, 16, 15, 19, 12, 18, 4, 9, 6)_{\Theta(3,5,7)},$
 $(21, 0, 5, 18, 11, 7, 9, 22, 13, 8, 14, 1, 20, 24)_{\Theta(3,5,7)},$
 $(0, 22, 13, 4, 5, 6, 19, 23, 12, 21, 8, 2, 1, 10)_{\Theta(3,6,6)},$
 $(2, 9, 3, 4, 18, 11, 1, 12, 16, 23, 5, 8, 19, 0)_{\Theta(3,6,6)},$
 $(1, 6, 7, 0, 9, 20, 10, 21, 23, 17, 22, 19, 2, 4)_{\Theta(3,6,6)},$
 $(9, 5, 19, 13, 6, 1, 23, 8, 3, 10, 14, 2, 0, 22)_{\Theta(3,6,6)},$
 $(0, 1, 22, 15, 8, 11, 21, 12, 19, 5, 14, 18, 20, 3)_{\Theta(4,4,7)},$
 $(0, 16, 5, 4, 14, 17, 18, 15, 21, 1, 23, 6, 2, 19)_{\Theta(4,4,7)},$
 $(4, 12, 22, 17, 11, 21, 14, 3, 13, 0, 6, 18, 8, 9)_{\Theta(4,4,7)},$
 $(19, 7, 17, 23, 3, 6, 15, 5, 14, 8, 22, 12, 24, 20)_{\Theta(4,4,7)},$
 $(0, 1, 17, 22, 19, 9, 13, 5, 23, 14, 12, 18, 2, 11)_{\Theta(4,5,6)},$
 $(13, 22, 21, 23, 18, 14, 6, 19, 4, 15, 2, 5, 20, 8)_{\Theta(4,5,6)},$
 $(13, 7, 19, 3, 21, 10, 4, 9, 6, 23, 22, 20, 15, 1)_{\Theta(4,5,6)},$
 $(1, 5, 10, 9, 11, 22, 14, 2, 12, 21, 0, 4, 18, 6)_{\Theta(4,5,6)},$
 $(0, 2, 17, 20, 19, 7, 4, 18, 21, 23, 11, 9, 13, 15)_{\Theta(5,5,5)},$
 $(0, 1, 16, 3, 20, 5, 13, 10, 12, 19, 6, 23, 18, 7)_{\Theta(5,5,5)},$
 $(20, 2, 14, 22, 6, 16, 13, 7, 23, 4, 21, 9, 19, 3)_{\Theta(5,5,5)},$
 $(0, 21, 5, 12, 22, 1, 14, 19, 18, 3, 9, 6, 7, 4)_{\Theta(5,5,5)}$

under the action of the mapping $x \mapsto x + 5 \pmod{25}$.

K_{36} Let the vertex set be $Z_{35} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 30, 14, 26, 7, 10, 4, 23, 12, 21, 34, 31, 33, 18)_{\Theta(1,2,12)},$
 $(7, 29, 19, 34, 1, 8, 5, 3, 25, 4, 15, 11, 31, 30)_{\Theta(1,2,12)},$
 $(16, 12, 2, 22, 8, 29, 25, 5, 33, 6, 23, 26, 0, 10)_{\Theta(1,2,12)},$
 $(5, 10, 22, 32, 2, 30, 16, 0, 11, 19, 24, 23, 33, 28)_{\Theta(1,2,12)},$
 $(18, 24, 27, 9, 2, 26, 20, 7, 3, 34, 11, 16, 28, 22)_{\Theta(1,2,12)},$
 $(19, 2, 1, 4, 11, 21, 8, 0, 9, 33, 17, 18, \infty, 22)_{\Theta(1,2,12)},$
 $(\infty, 14, 27, 5, 19, 1, 28, 33, 0, 3, 34, 18, 24, 22)_{\Theta(1,4,10)},$
 $(24, 12, 30, 10, 34, 13, 5, 28, 32, 3, 6, 4, 0, 2)_{\Theta(1,4,10)},$
 $(24, 11, 15, 31, 18, 27, 26, 30, 20, 13, 3, 12, 23, 6)_{\Theta(1,4,10)},$
 $(9, 26, 27, 15, 32, 1, 0, 21, 17, 18, 33, 24, 25, 20)_{\Theta(1,4,10)},$
 $(4, 18, 20, 12, 32, 11, 0, 7, 26, 29, 22, 1, 27, 16)_{\Theta(1,4,10)},$
 $(15, 6, 28, 16, 26, 12, 7, 23, 5, \infty, 3, 4, 14, 29)_{\Theta(1,4,10)},$
 $(\infty, 28, 7, 25, 17, 20, 15, 14, 2, 1, 31, 24, 18, 21)_{\Theta(1,6,8)},$
 $(9, 2, 20, 18, 31, 0, 21, 8, 26, 25, 16, 1, 28, 27)_{\Theta(1,6,8)},$
 $(17, 13, 14, 31, 33, 29, 25, 3, 8, 2, 26, 12, 23, 34)_{\Theta(1,6,8)},$
 $(5, 16, 15, 21, 34, 0, 19, 14, 24, 1, 32, 2, 25, 28)_{\Theta(1,6,8)},$
 $(23, 4, 15, 22, 31, 29, 2, 8, 34, 17, 11, 30, 10, 24)_{\Theta(1,6,8)},$
 $(27, 14, 5, 7, 22, 3, 20, 18, 8, 15, \infty, 1, 11, 19)_{\Theta(1,6,8)},$
 $(\infty, 11, 28, 6, 20, 15, 32, 17, 23, 13, 4, 21, 9, 19)_{\Theta(2,2,11)},$
 $(18, 11, 21, 14, 4, 17, 31, 3, 22, 24, 0, 15, 13, 33)_{\Theta(2,2,11)},$
 $(19, 4, 0, 26, 13, 11, 3, 14, 29, 24, 30, 33, 2, 12)_{\Theta(2,2,11)},$
 $(34, 0, 1, 18, 33, 7, 10, 26, 15, 3, 17, 30, 20, 13)_{\Theta(2,2,11)},$

$(24, 12, 10, 17, 27, 15, 14, 5, 1, 32, 16, 25, 31, 11)_{\Theta(2,2,11)},$
 $(1, 24, 7, 12, 13, 2, 3, 8, 0, 27, 20, 6, 32, \infty)_{\Theta(2,2,11)},$
 $(\infty, 28, 14, 3, 12, 25, 22, 11, 8, 15, 31, 23, 29, 4)_{\Theta(2,3,10)},$
 $(24, 8, 28, 32, 20, 23, 12, 34, 16, 14, 15, 5, 6, 26)_{\Theta(2,3,10)},$
 $(25, 17, 14, 30, 22, 33, 8, 3, 31, 19, 34, 29, 12, 11)_{\Theta(2,3,10)},$
 $(12, 17, 27, 16, 24, 21, 15, 9, 7, 26, 31, 5, 19, 10)_{\Theta(2,3,10)},$
 $(6, 5, 20, 34, 22, 19, 0, 11, 33, 32, 3, 17, 13, 1)_{\Theta(2,3,10)},$
 $(18, 5, 9, 0, 3, 16, 2, \infty, 1, 11, 14, 33, 20, 7)_{\Theta(2,3,10)},$
 $(\infty, 10, 1, 25, 5, 19, 22, 18, 11, 30, 2, 27, 9, 32)_{\Theta(2,4,9)},$
 $(33, 0, 28, 22, 14, 13, 19, 34, 31, 8, 6, 17, 3, 12)_{\Theta(2,4,9)},$
 $(27, 30, 25, 8, 7, 22, 10, 9, 19, 1, 34, 28, 24, 20)_{\Theta(2,4,9)},$
 $(33, 27, 11, 14, 25, 31, 16, 30, 24, 13, 28, 22, 9, 1)_{\Theta(2,4,9)},$
 $(12, 18, 9, 15, 16, 8, 26, 30, 28, 5, 24, 29, 6, 21)_{\Theta(2,4,9)},$
 $(29, 6, 1, 16, 17, 12, 22, 24, \infty, 8, 0, 3, 20, 31)_{\Theta(2,4,9)},$
 $(\infty, 34, 15, 29, 9, 5, 32, 18, 26, 20, 33, 28, 2, 6)_{\Theta(2,5,8)},$
 $(8, 18, 24, 10, 19, 29, 31, 32, 27, 16, 17, 0, 7, 28)_{\Theta(2,5,8)},$
 $(3, 7, 26, 21, 23, 0, 24, 34, 29, 8, 15, 30, 20, 1)_{\Theta(2,5,8)},$
 $(20, 32, 11, 21, 28, 13, 10, 17, 7, 3, 2, 8, 0, 29)_{\Theta(2,5,8)},$
 $(12, 29, 28, 21, 18, 0, 2, 32, 9, 30, 16, 34, 26, 6)_{\Theta(2,5,8)},$
 $(29, 1, 26, 3, 14, 7, \infty, 30, 0, 12, 34, 21, 10, 6)_{\Theta(2,5,8)},$
 $(\infty, 22, 32, 11, 28, 6, 29, 19, 15, 25, 24, 7, 2, 33)_{\Theta(2,6,7)},$
 $(6, 9, 26, 20, 11, 2, 4, 0, 5, 12, 3, 8, 15, 17)_{\Theta(2,6,7)},$
 $(0, 34, 18, 30, 24, 31, 32, 12, 16, 22, 10, 21, 19, 23)_{\Theta(2,6,7)},$
 $(1, 27, 11, 9, 24, 29, 17, 30, 12, 18, 32, 5, 3, 6)_{\Theta(2,6,7)},$
 $(4, 8, 1, 3, 29, 16, 21, 33, 25, 13, 10, 30, 14, 7)_{\Theta(2,6,7)},$
 $(2, 26, 18, 6, 0, 8, 14, 28, 20, 33, 13, \infty, 19, 30)_{\Theta(2,6,7)},$
 $(\infty, 11, 8, 4, 6, 7, 3, 32, 5, 23, 25, 15, 30, 26)_{\Theta(3,4,8)},$
 $(20, 28, 15, 16, 33, 27, 6, 13, 29, 17, 34, 14, 1, 4)_{\Theta(3,4,8)},$
 $(4, 6, 12, 17, 0, 23, 8, 16, 10, 31, 14, 7, 5, 13)_{\Theta(3,4,8)},$
 $(7, 24, 23, 33, 4, 31, 22, 28, 2, 17, 1, 18, 13, 19)_{\Theta(3,4,8)},$
 $(31, 9, 6, 11, 20, 2, 25, 27, 17, 18, 19, 29, 15, 22)_{\Theta(3,4,8)},$
 $(29, 4, 0, \infty, 30, 21, 18, 20, 1, 7, 10, 13, 2, 15)_{\Theta(3,4,8)},$
 $(\infty, 19, 21, 32, 34, 31, 0, 14, 17, 5, 10, 7, 12, 30)_{\Theta(3,6,6)},$
 $(15, 22, 17, 10, 0, 29, 28, 12, 8, 25, 6, 30, 21, 16)_{\Theta(3,6,6)},$
 $(25, 14, 11, 34, 9, 1, 23, 15, 8, 17, 16, 29, 2, 28)_{\Theta(3,6,6)},$
 $(12, 6, 2, 13, 25, 28, 15, 19, 21, 32, 1, 26, 20, 8)_{\Theta(3,6,6)},$
 $(27, 24, 28, 8, 1, 19, 12, 31, 13, 9, 4, 5, 6, 33)_{\Theta(3,6,6)},$
 $(10, 3, 8, 13, 19, 9, 16, 2, \infty, 28, 24, 12, 18, 6)_{\Theta(3,6,6)},$
 $(\infty, 28, 29, 27, 4, 16, 15, 5, 13, 30, 19, 12, 2, 18)_{\Theta(4,4,7)},$
 $(27, 10, 9, 12, 3, 16, 5, 1, 19, 28, 29, 6, 25, 4)_{\Theta(4,4,7)},$
 $(17, 30, 25, 2, 16, 21, 18, 10, 3, 27, 20, 7, 26, 32)_{\Theta(4,4,7)},$
 $(31, 26, 6, 13, 28, 29, 16, 3, 1, 30, 34, 24, 4, 8)_{\Theta(4,4,7)},$
 $(30, 26, 25, 34, 29, 27, 22, 9, 12, 21, 6, 33, 20, 19)_{\Theta(4,4,7)},$
 $(17, 4, 2, 3, 33, 23, 20, 18, 13, 29, 10, \infty, 32, 31)_{\Theta(4,4,7)},$
 $(\infty, 21, 9, 7, 27, 11, 5, 31, 20, 2, 34, 23, 26, 13)_{\Theta(4,5,6)},$

$(29, 5, 24, 14, 15, 28, 32, 1, 17, 3, 13, 25, 11, 12)_{\Theta(4,5,6)},$
 $(23, 34, 9, 0, 22, 21, 33, 13, 7, 29, 11, 18, 17, 3)_{\Theta(4,5,6)},$
 $(26, 28, 19, 0, 20, 11, 24, 1, 12, 10, 34, 31, 4, 25)_{\Theta(4,5,6)},$
 $(1, 23, 31, 22, 32, 11, 28, 15, 12, 5, 9, 29, 0, 18)_{\Theta(4,5,6)},$
 $(9, 2, 22, 5, 0, 11, 32, 14, 7, 28, 30, 23, \infty, 10)_{\Theta(4,5,6)}$

under the action of the mapping $x \mapsto x + 5 \pmod{35}$, $\infty \mapsto \infty$.

K_{40} Let the vertex set be $Z_{39} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 3, 37, 29, 1, 25, 32, 5, 34, 13, 26, 23, 21, 19)_{\Theta(1,2,12)},$
 $(5, 24, 38, 4, 8, 0, 31, 14, 22, 12, 6, 11, 29, 3)_{\Theta(1,2,12)},$
 $(8, 32, 36, 38, 22, 15, 18, 1, 27, 0, 29, 10, 16, 7)_{\Theta(1,2,12)},$
 $(13, 27, 16, 1, 36, 35, 3, 4, 2, 18, 9, 26, 31, 12)_{\Theta(1,2,12)},$
 $(\infty, 1, 15, 30, 5, 29, 26, 10, 7, 28, 35, 12, 11, 21)_{\Theta(1,4,10)},$
 $(25, 36, 5, 17, 2, 38, 10, 35, 4, 30, 32, 34, 0, 6)_{\Theta(1,4,10)},$
 $(20, 27, 38, 5, 14, 0, 25, 13, 7, 6, 28, 21, 24, 35)_{\Theta(1,4,10)},$
 $(25, 15, 35, 1, 31, 10, 6, 2, 19, 20, 3, 21, 9, 7)_{\Theta(1,4,10)},$
 $(\infty, 2, 34, 32, 1, 9, 35, 33, 14, 18, 37, 16, 5, 30)_{\Theta(1,6,8)},$
 $(20, 13, 29, 27, 4, 38, 0, 5, 36, 9, 10, 34, 31, 24)_{\Theta(1,6,8)},$
 $(25, 3, 19, 33, 31, 36, 18, 21, 30, 35, 10, 11, 32, 0)_{\Theta(1,6,8)},$
 $(10, 14, 37, 17, 27, 33, 4, 19, 2, 5, 31, 8, 20, 36)_{\Theta(1,6,8)},$
 $(\infty, 10, 16, 9, 5, 37, 28, 1, 25, 3, 13, 2, 36, 30)_{\Theta(2,2,11)},$
 $(0, 19, 27, 30, 38, 12, 28, 15, 29, 14, 10, 26, 13, 5)_{\Theta(2,2,11)},$
 $(10, 30, 6, 28, 7, 12, 15, 8, 37, 32, 29, 9, 11, 38)_{\Theta(2,2,11)},$
 $(14, 27, 34, 23, 24, 3, 20, 22, 36, 8, 25, 26, 5, 11)_{\Theta(2,2,11)},$
 $(\infty, 13, 27, 19, 6, 38, 16, 28, 12, 7, 32, 1, 0, 20)_{\Theta(2,3,10)},$
 $(15, 16, 5, 11, 6, 34, 14, 38, 24, 35, 31, 22, 23, 26)_{\Theta(2,3,10)},$
 $(26, 0, 31, 13, 9, 10, 25, 22, 4, 15, 2, 32, 20, 38)_{\Theta(2,3,10)},$
 $(33, 1, 18, 21, 3, 36, 5, 12, 35, 2, 24, 30, 32, 34)_{\Theta(2,3,10)},$
 $(\infty, 25, 0, 16, 15, 13, 26, 5, 21, 30, 8, 38, 18, 36)_{\Theta(2,4,9)},$
 $(19, 15, 17, 24, 34, 3, 10, 28, 2, 33, 20, 35, 21, 14)_{\Theta(2,4,9)},$
 $(25, 16, 20, 17, 37, 1, 18, 14, 2, 35, 10, 36, 30, 33)_{\Theta(2,4,9)},$
 $(2, 19, 13, 1, 8, 3, 5, 15, 0, 4, 20, 37, 18, 29)_{\Theta(2,4,9)},$
 $(\infty, 11, 4, 6, 2, 22, 38, 32, 14, 9, 17, 31, 7, 16)_{\Theta(2,5,8)},$
 $(22, 15, 35, 30, 31, 32, 26, 0, 12, 1, 28, 21, 25, 9)_{\Theta(2,5,8)},$
 $(1, 2, 37, 27, 3, 35, 18, 23, 34, 32, 33, 12, 26, 0)_{\Theta(2,5,8)},$
 $(26, 22, 16, 29, 5, 14, 24, 34, 0, 9, 6, 25, 7, 36)_{\Theta(2,5,8)},$
 $(\infty, 38, 3, 19, 35, 24, 26, 18, 17, 29, 25, 8, 11, 32)_{\Theta(2,6,7)},$
 $(15, 30, 20, 2, 3, 10, 33, 21, 28, 37, 18, 7, 4, 31)_{\Theta(2,6,7)},$
 $(5, 34, 35, 13, 38, 19, 15, 12, 21, 14, 16, 22, 24, 9)_{\Theta(2,6,7)},$
 $(0, 26, 31, 17, 4, 14, 7, 12, 21, 15, 25, 1, 22, 11)_{\Theta(2,6,7)},$
 $(\infty, 33, 21, 8, 13, 17, 26, 32, 31, 27, 4, 1, 25, 3)_{\Theta(3,4,8)},$
 $(22, 19, 31, 8, 4, 38, 9, 5, 20, 18, 25, 27, 7, 32)_{\Theta(3,4,8)},$
 $(31, 23, 4, 35, 29, 24, 13, 38, 15, 12, 30, 11, 0, 17)_{\Theta(3,4,8)},$
 $(37, 34, 17, 9, 4, 12, 0, 24, 20, 23, 2, 3, 27, 33)_{\Theta(3,4,8)},$
 $(\infty, 29, 33, 17, 38, 13, 5, 8, 0, 16, 4, 7, 11, 15)_{\Theta(3,6,6)},$
 $(25, 7, 26, 15, 35, 1, 12, 36, 16, 14, 8, 27, 5, 29)_{\Theta(3,6,6)},$

$(8, 5, 31, 18, 26, 27, 10, 17, 12, 28, 30, 25, 9, 3)_{\Theta(3,6,6)},$
 $(21, 31, 25, 7, 12, 22, 15, 18, 6, 3, 4, 2, 11, 37)_{\Theta(3,6,6)},$
 $(\infty, 7, 30, 13, 22, 14, 16, 32, 4, 17, 28, 24, 3, 8)_{\Theta(4,4,7)},$
 $(13, 32, 34, 29, 22, 6, 7, 12, 27, 30, 5, 26, 17, 15)_{\Theta(4,4,7)},$
 $(16, 20, 6, 34, 37, 3, 30, 23, 22, 10, 2, 12, 11, 5)_{\Theta(4,4,7)},$
 $(5, 6, 25, 9, 0, 36, 8, 4, 17, 30, 10, 18, 33, 29)_{\Theta(4,4,7)}$
 $(\infty, 23, 25, 31, 1, 27, 18, 29, 16, 14, 34, 19, 6, 24)_{\Theta(4,5,6)},$
 $(11, 16, 21, 17, 13, 23, 9, 33, 36, 27, 31, 30, 37, 26)_{\Theta(4,5,6)},$
 $(8, 2, 14, 33, 16, 21, 7, 28, 0, 26, 9, 36, 5, 10)_{\Theta(4,5,6)},$
 $(32, 12, 23, 22, 10, 16, 0, 5, 7, 17, 20, 27, 33, 4)_{\Theta(4,5,6)}$

under the action of the mapping $x \mapsto x + 3 \pmod{39}$, $\infty \mapsto \infty$.

K_{51} Let the vertex set be Z_{51} . The decompositions consist of the graphs

$(0, 47, 18, 49, 16, 38, 26, 27, 46, 33, 9, 48, 5, 36)_{\Theta(1,2,12)},$
 $(24, 15, 8, 9, 23, 44, 19, 17, 27, 10, 14, 46, 43, 2)_{\Theta(1,2,12)},$
 $(30, 35, 19, 5, 33, 14, 28, 21, 22, 27, 24, 16, 37, 44)_{\Theta(1,2,12)},$
 $(0, 25, 45, 37, 21, 42, 44, 31, 40, 17, 23, 28, 11, 26)_{\Theta(1,2,12)},$
 $(45, 22, 11, 49, 41, 38, 5, 32, 1, 7, 19, 9, 31, 46)_{\Theta(1,2,12)},$
 $(0, 38, 25, 13, 16, 10, 44, 26, 5, 33, 40, 47, 27, 24)_{\Theta(1,4,10)},$
 $(41, 38, 39, 37, 1, 10, 24, 40, 32, 6, 48, 8, 49, 22)_{\Theta(1,4,10)},$
 $(16, 48, 3, 42, 18, 44, 43, 1, 46, 25, 30, 11, 22, 26)_{\Theta(1,4,10)},$
 $(13, 42, 21, 11, 27, 26, 35, 18, 23, 8, 10, 6, 24, 25)_{\Theta(1,4,10)},$
 $(39, 47, 19, 42, 35, 28, 2, 3, 9, 5, 11, 43, 25, 20)_{\Theta(1,4,10)},$
 $(0, 38, 4, 48, 31, 17, 34, 20, 1, 42, 22, 14, 33, 44)_{\Theta(1,6,8)},$
 $(5, 30, 33, 23, 9, 38, 29, 16, 28, 13, 31, 1, 36, 4)_{\Theta(1,6,8)},$
 $(9, 39, 2, 45, 43, 16, 25, 6, 30, 19, 27, 28, 15, 48)_{\Theta(1,6,8)},$
 $(47, 11, 37, 15, 17, 44, 16, 40, 12, 6, 45, 9, 14, 49)_{\Theta(1,6,8)},$
 $(5, 23, 35, 4, 10, 8, 11, 9, 26, 1, 2, 31, 34, 39)_{\Theta(1,6,8)},$
 $(0, 42, 18, 11, 44, 10, 12, 27, 4, 33, 23, 48, 13, 8)_{\Theta(2,2,11)},$
 $(43, 44, 7, 40, 49, 42, 23, 11, 1, 18, 47, 46, 2, 29)_{\Theta(2,2,11)},$
 $(22, 43, 1, 35, 40, 48, 45, 2, 30, 18, 31, 42, 26, 24)_{\Theta(2,2,11)},$
 $(38, 11, 13, 44, 42, 21, 8, 28, 47, 17, 3, 4, 9, 46)_{\Theta(2,2,11)},$
 $(40, 8, 17, 9, 29, 26, 4, 16, 6, 0, 25, 49, 45, 3)_{\Theta(2,2,11)},$
 $(0, 42, 19, 26, 47, 22, 27, 17, 7, 34, 23, 16, 46, 38)_{\Theta(2,3,10)},$
 $(0, 46, 8, 13, 12, 24, 33, 7, 40, 41, 10, 19, 39, 31)_{\Theta(2,3,10)},$
 $(20, 45, 38, 42, 31, 39, 21, 5, 1, 46, 34, 36, 43, 33)_{\Theta(2,3,10)},$
 $(10, 39, 26, 6, 36, 32, 37, 34, 11, 17, 18, 3, 14, 2)_{\Theta(2,3,10)},$
 $(41, 3, 5, 7, 26, 43, 17, 0, 6, 22, 8, 35, 32, 23)_{\Theta(2,3,10)},$
 $(0, 44, 7, 3, 42, 21, 24, 29, 49, 10, 46, 16, 17, 9)_{\Theta(2,4,9)},$
 $(46, 36, 37, 30, 28, 18, 2, 21, 38, 24, 19, 35, 29, 40)_{\Theta(2,4,9)},$
 $(20, 19, 0, 42, 29, 32, 47, 36, 49, 22, 30, 5, 6, 37)_{\Theta(2,4,9)},$
 $(24, 5, 7, 30, 1, 35, 39, 28, 20, 16, 26, 33, 29, 34)_{\Theta(2,4,9)},$
 $(18, 32, 20, 9, 46, 14, 8, 34, 6, 31, 25, 28, 5, 41)_{\Theta(2,4,9)},$
 $(0, 16, 8, 15, 27, 13, 9, 48, 3, 37, 6, 4, 1, 31)_{\Theta(2,5,8)},$
 $(27, 18, 8, 20, 22, 40, 2, 9, 35, 24, 38, 23, 43, 42)_{\Theta(2,5,8)},$
 $(12, 35, 33, 17, 41, 21, 49, 7, 31, 18, 10, 4, 30, 39)_{\Theta(2,5,8)},$

$(43, 40, 8, 27, 16, 26, 17, 4, 36, 35, 31, 14, 32, 11)_{\Theta(2,5,8)},$
 $(44, 43, 38, 19, 9, 31, 32, 6, 23, 11, 14, 36, 8, 1)_{\Theta(2,5,8)},$
 $(0, 11, 39, 36, 16, 28, 14, 29, 18, 47, 7, 13, 26, 19)_{\Theta(2,6,7)},$
 $(9, 16, 13, 2, 15, 10, 45, 41, 17, 34, 5, 40, 4, 37)_{\Theta(2,6,7)},$
 $(3, 12, 9, 45, 15, 39, 38, 37, 14, 48, 10, 8, 29, 1)_{\Theta(2,6,7)},$
 $(12, 39, 49, 34, 29, 20, 18, 37, 13, 36, 41, 10, 14, 46)_{\Theta(2,6,7)},$
 $(40, 32, 48, 6, 38, 44, 3, 29, 13, 4, 14, 0, 20, 8)_{\Theta(2,6,7)},$
 $(0, 40, 33, 39, 34, 13, 18, 20, 12, 36, 41, 42, 45, 43)_{\Theta(3,4,8)},$
 $(49, 10, 7, 32, 30, 40, 23, 6, 27, 29, 28, 15, 8, 26)_{\Theta(3,4,8)},$
 $(32, 10, 0, 14, 44, 15, 2, 46, 18, 34, 27, 1, 19, 17)_{\Theta(3,4,8)},$
 $(24, 41, 33, 45, 47, 6, 10, 9, 35, 16, 44, 38, 2, 11)_{\Theta(3,4,8)},$
 $(10, 34, 37, 22, 21, 38, 3, 4, 14, 17, 28, 42, 2, 29)_{\Theta(3,4,8)},$
 $(0, 19, 22, 32, 26, 21, 30, 2, 31, 29, 48, 38, 24, 45)_{\Theta(3,6,6)},$
 $(34, 40, 2, 7, 3, 19, 35, 33, 43, 32, 31, 1, 24, 17)_{\Theta(3,6,6)},$
 $(46, 16, 9, 20, 32, 8, 23, 26, 5, 12, 36, 48, 44, 27)_{\Theta(3,6,6)},$
 $(36, 35, 4, 0, 43, 37, 45, 40, 27, 5, 6, 24, 9, 10)_{\Theta(3,6,6)},$
 $(17, 16, 5, 47, 11, 28, 19, 26, 8, 30, 24, 27, 25, 1)_{\Theta(3,6,6)},$
 $(0, 9, 16, 3, 15, 27, 48, 46, 17, 24, 11, 10, 4, 28)_{\Theta(4,4,7)},$
 $(10, 24, 29, 0, 8, 39, 21, 44, 18, 9, 40, 17, 42, 19)_{\Theta(4,4,7)},$
 $(13, 32, 5, 11, 8, 47, 34, 44, 9, 23, 2, 35, 49, 16)_{\Theta(4,4,7)},$
 $(0, 12, 1, 45, 48, 11, 26, 17, 32, 37, 41, 34, 25, 22)_{\Theta(4,4,7)},$
 $(6, 26, 8, 18, 1, 5, 7, 37, 2, 22, 34, 19, 30, 4)_{\Theta(4,4,7)},$
 $(0, 28, 26, 35, 2, 13, 27, 22, 46, 38, 41, 37, 30, 9)_{\Theta(4,5,6)},$
 $(9, 35, 21, 45, 14, 12, 20, 26, 11, 13, 39, 2, 25, 4)_{\Theta(4,5,6)},$
 $(2, 4, 13, 21, 5, 3, 47, 40, 23, 16, 33, 27, 45, 46)_{\Theta(4,5,6)},$
 $(13, 48, 5, 45, 25, 1, 16, 27, 44, 19, 9, 14, 2, 12)_{\Theta(4,5,6)},$
 $(13, 33, 23, 10, 5, 16, 0, 2, 4, 35, 6, 15, 47, 31)_{\Theta(4,5,6)}$

under the action of the mapping $x \mapsto x + 3 \pmod{51}$.

K_{55} Let the vertex set be Z_{55} . The decompositions consist of the graphs

$(0, 29, 41, 26, 15, 13, 44, 50, 28, 34, 32, 4, 22, 45)_{\Theta(1,2,12)},$
 $(8, 44, 31, 7, 48, 43, 20, 28, 0, 42, 49, 37, 34, 9)_{\Theta(1,2,12)},$
 $(21, 39, 18, 19, 15, 30, 11, 27, 49, 16, 40, 53, 43, 50)_{\Theta(1,2,12)},$
 $(49, 32, 1, 39, 30, 20, 26, 48, 34, 18, 35, 47, 46, 37)_{\Theta(1,2,12)},$
 $(37, 53, 28, 15, 36, 4, 40, 19, 50, 22, 5, 51, 6, 2)_{\Theta(1,2,12)},$
 $(35, 27, 38, 10, 53, 11, 19, 23, 6, 26, 30, 29, 34, 8)_{\Theta(1,2,12)},$
 $(17, 43, 51, 11, 10, 7, 13, 28, 46, 16, 31, 36, 47, 23)_{\Theta(1,2,12)},$
 $(27, 52, 42, 45, 25, 39, 11, 47, 12, 4, 17, 31, 14, 29)_{\Theta(1,2,12)},$
 $(51, 53, 35, 3, 49, 46, 17, 10, 5, 7, 28, 16, 43, 54)_{\Theta(1,2,12)},$
 $(0, 8, 16, 17, 35, 36, 19, 44, 4, 10, 15, 22, 33, 26)_{\Theta(1,4,10)},$
 $(52, 53, 10, 20, 22, 33, 31, 49, 4, 50, 14, 19, 42, 29)_{\Theta(1,4,10)},$
 $(24, 4, 11, 46, 23, 8, 45, 44, 53, 3, 6, 36, 17, 12)_{\Theta(1,4,10)},$
 $(53, 0, 26, 40, 12, 36, 2, 47, 19, 11, 39, 41, 31, 23)_{\Theta(1,4,10)},$
 $(28, 49, 45, 15, 16, 2, 48, 23, 43, 1, 47, 51, 30, 6)_{\Theta(1,4,10)},$
 $(13, 7, 47, 41, 45, 10, 53, 9, 35, 50, 34, 52, 38, 23)_{\Theta(1,4,10)},$
 $(0, 11, 13, 3, 4, 26, 52, 48, 19, 17, 6, 22, 25, 16)_{\Theta(1,4,10)},$

$(5, 49, 25, 2, 26, 27, 42, 4, 8, 41, 35, 13, 19, 15)_{\Theta(1,4,10)},$
 $(13, 1, 20, 12, 42, 54, 2, 9, 50, 19, 52, 32, 44, 41)_{\Theta(1,4,10)},$
 $(0, 28, 39, 26, 18, 31, 51, 37, 3, 49, 9, 46, 12, 38)_{\Theta(1,6,8)},$
 $(3, 17, 41, 38, 31, 30, 50, 35, 52, 49, 29, 13, 32, 34)_{\Theta(1,6,8)},$
 $(9, 14, 3, 15, 13, 1, 25, 16, 53, 23, 34, 6, 12, 39)_{\Theta(1,6,8)},$
 $(26, 42, 22, 29, 25, 44, 12, 34, 1, 40, 11, 38, 35, 41)_{\Theta(1,6,8)},$
 $(31, 6, 17, 8, 13, 46, 37, 35, 10, 7, 26, 15, 2, 44)_{\Theta(1,6,8)},$
 $(21, 40, 0, 49, 2, 13, 33, 26, 28, 12, 34, 5, 20, 3)_{\Theta(1,6,8)},$
 $(3, 18, 7, 22, 4, 25, 17, 29, 33, 27, 1, 15, 10, 42)_{\Theta(1,6,8)},$
 $(7, 52, 50, 8, 9, 10, 17, 5, 38, 4, 18, 49, 51, 41)_{\Theta(1,6,8)},$
 $(40, 50, 9, 28, 20, 29, 41, 54, 31, 46, 49, 39, 27, 22)_{\Theta(1,6,8)},$
 $(0, 16, 53, 3, 43, 44, 13, 37, 18, 51, 4, 10, 25, 11)_{\Theta(2,2,11)},$
 $(29, 38, 10, 37, 48, 19, 9, 11, 20, 50, 40, 33, 8, 28)_{\Theta(2,2,11)},$
 $(52, 21, 7, 40, 16, 10, 49, 27, 20, 24, 36, 29, 50, 18)_{\Theta(2,2,11)},$
 $(26, 43, 47, 22, 2, 7, 13, 0, 51, 42, 39, 34, 45, 25)_{\Theta(2,2,11)},$
 $(18, 25, 26, 24, 29, 14, 7, 30, 51, 31, 4, 43, 39, 52)_{\Theta(2,2,11)},$
 $(22, 7, 6, 20, 48, 41, 47, 38, 53, 15, 24, 33, 0, 37)_{\Theta(2,2,11)},$
 $(2, 50, 17, 19, 22, 51, 23, 6, 30, 1, 31, 20, 15, 12)_{\Theta(2,2,11)},$
 $(23, 9, 46, 7, 28, 26, 37, 19, 42, 53, 41, 51, 34, 31)_{\Theta(2,2,11)},$
 $(35, 36, 51, 49, 43, 9, 29, 0, 47, 19, 7, 48, 34, 4)_{\Theta(2,2,11)},$
 $(0, 20, 1, 35, 7, 10, 13, 14, 41, 8, 2, 9, 43, 45)_{\Theta(2,3,10)},$
 $(14, 45, 27, 46, 22, 32, 23, 33, 53, 24, 25, 30, 1, 38)_{\Theta(2,3,10)},$
 $(33, 12, 26, 1, 53, 10, 48, 46, 24, 0, 22, 34, 17, 38)_{\Theta(2,3,10)},$
 $(22, 34, 53, 16, 43, 14, 4, 50, 12, 10, 26, 37, 18, 23)_{\Theta(2,3,10)},$
 $(7, 46, 15, 4, 51, 34, 50, 24, 45, 23, 27, 6, 9, 5)_{\Theta(2,3,10)},$
 $(19, 52, 36, 42, 53, 13, 0, 51, 49, 9, 16, 29, 15, 26)_{\Theta(2,3,10)},$
 $(52, 50, 13, 19, 1, 45, 39, 43, 29, 10, 31, 11, 2, 7)_{\Theta(2,3,10)},$
 $(28, 44, 0, 53, 13, 41, 26, 16, 25, 10, 7, 27, 29, 24)_{\Theta(2,3,10)},$
 $(41, 47, 37, 16, 17, 29, 4, 43, 0, 8, 46, 38, 26, 7)_{\Theta(2,3,10)},$
 $(0, 28, 17, 6, 51, 7, 4, 12, 8, 40, 39, 42, 16, 38)_{\Theta(2,4,9)},$
 $(50, 49, 46, 42, 48, 8, 39, 32, 12, 16, 13, 15, 17, 11)_{\Theta(2,4,9)},$
 $(53, 41, 19, 12, 14, 32, 23, 36, 6, 20, 33, 30, 45, 15)_{\Theta(2,4,9)},$
 $(1, 4, 14, 24, 12, 44, 3, 51, 30, 37, 21, 13, 48, 11)_{\Theta(2,4,9)},$
 $(4, 14, 41, 28, 1, 6, 9, 25, 53, 49, 19, 0, 31, 40)_{\Theta(2,4,9)},$
 $(30, 33, 17, 42, 10, 28, 13, 39, 48, 4, 23, 15, 29, 2)_{\Theta(2,4,9)},$
 $(8, 53, 7, 9, 42, 27, 24, 45, 17, 35, 29, 5, 10, 21)_{\Theta(2,4,9)},$
 $(8, 20, 30, 46, 11, 42, 14, 49, 40, 37, 7, 26, 25, 13)_{\Theta(2,4,9)},$
 $(17, 37, 16, 34, 46, 32, 27, 8, 51, 11, 30, 10, 26, 24)_{\Theta(2,4,9)},$
 $(0, 33, 11, 5, 37, 18, 38, 51, 48, 1, 17, 35, 36, 22)_{\Theta(2,5,8)},$
 $(34, 35, 48, 38, 14, 7, 29, 37, 9, 5, 53, 24, 45, 52)_{\Theta(2,5,8)},$
 $(24, 2, 10, 3, 35, 23, 21, 1, 28, 37, 29, 47, 26, 43)_{\Theta(2,5,8)},$
 $(13, 29, 12, 31, 27, 36, 44, 42, 37, 17, 47, 21, 45, 19)_{\Theta(2,5,8)},$
 $(16, 5, 11, 30, 45, 14, 26, 19, 1, 29, 20, 9, 8, 25)_{\Theta(2,5,8)},$
 $(3, 14, 9, 51, 36, 29, 30, 33, 21, 4, 23, 17, 41, 50)_{\Theta(2,5,8)},$
 $(6, 18, 31, 17, 27, 5, 2, 26, 39, 7, 3, 48, 4, 34)_{\Theta(2,5,8)},$
 $(48, 7, 50, 15, 23, 46, 20, 27, 14, 16, 6, 39, 4, 47)_{\Theta(2,5,8)},$

$(45, 3, 0, 20, 36, 42, 18, 8, 35, 37, 10, 46, 47, 49)_{\Theta(2,5,8)},$
 $(0, 28, 48, 44, 51, 2, 41, 43, 26, 14, 7, 21, 24, 6)_{\Theta(2,6,7)},$
 $(43, 8, 36, 25, 15, 22, 27, 52, 33, 21, 35, 19, 20, 44)_{\Theta(2,6,7)},$
 $(2, 44, 6, 31, 51, 36, 15, 40, 42, 30, 22, 9, 35, 14)_{\Theta(2,6,7)},$
 $(46, 26, 4, 37, 29, 39, 27, 10, 35, 49, 9, 32, 52, 23)_{\Theta(2,6,7)},$
 $(1, 27, 6, 9, 33, 4, 8, 16, 26, 49, 52, 15, 34, 43)_{\Theta(2,6,7)},$
 $(2, 50, 8, 15, 51, 52, 21, 30, 33, 45, 40, 0, 3, 26)_{\Theta(2,6,7)},$
 $(50, 51, 33, 47, 19, 24, 4, 38, 18, 45, 23, 29, 12, 49)_{\Theta(2,6,7)},$
 $(5, 44, 3, 27, 18, 10, 4, 43, 6, 0, 32, 30, 26, 16)_{\Theta(2,6,7)},$
 $(47, 53, 48, 20, 29, 7, 17, 36, 13, 24, 22, 8, 27, 23)_{\Theta(2,6,7)},$
 $(0, 6, 35, 40, 27, 42, 22, 1, 11, 18, 47, 10, 52, 50)_{\Theta(3,4,8)},$
 $(30, 32, 45, 35, 47, 17, 13, 8, 18, 29, 44, 49, 2, 26)_{\Theta(3,4,8)},$
 $(21, 40, 45, 39, 3, 29, 1, 46, 12, 23, 25, 16, 30, 9)_{\Theta(3,4,8)},$
 $(5, 17, 19, 26, 33, 53, 10, 37, 36, 44, 24, 11, 30, 38)_{\Theta(3,4,8)},$
 $(19, 40, 47, 24, 51, 23, 32, 15, 11, 46, 13, 20, 26, 7)_{\Theta(3,4,8)},$
 $(32, 16, 37, 19, 34, 33, 1, 46, 51, 22, 26, 13, 7, 8)_{\Theta(3,4,8)},$
 $(3, 13, 28, 45, 27, 49, 19, 53, 12, 24, 17, 33, 18, 0)_{\Theta(3,4,8)},$
 $(42, 46, 39, 29, 31, 43, 34, 30, 49, 53, 36, 14, 28, 9)_{\Theta(3,4,8)},$
 $(40, 24, 11, 8, 43, 9, 35, 10, 19, 32, 42, 4, 28, 26)_{\Theta(3,4,8)},$
 $(0, 39, 10, 29, 46, 18, 32, 53, 2, 49, 16, 20, 25, 41)_{\Theta(3,6,6)},$
 $(24, 19, 15, 13, 38, 14, 23, 40, 53, 9, 29, 7, 26, 50)_{\Theta(3,6,6)},$
 $(41, 12, 53, 52, 27, 10, 2, 28, 5, 33, 1, 26, 34, 9)_{\Theta(3,6,6)},$
 $(31, 11, 16, 42, 49, 20, 26, 37, 7, 32, 34, 39, 36, 0)_{\Theta(3,6,6)},$
 $(46, 35, 36, 53, 37, 16, 39, 1, 7, 25, 29, 50, 43, 13)_{\Theta(3,6,6)},$
 $(35, 53, 19, 8, 24, 23, 21, 28, 48, 17, 9, 38, 1, 14)_{\Theta(3,6,6)},$
 $(24, 50, 47, 27, 12, 2, 41, 34, 53, 28, 40, 48, 26, 23)_{\Theta(3,6,6)},$
 $(16, 28, 44, 17, 30, 10, 40, 42, 47, 51, 39, 22, 25, 37)_{\Theta(3,6,6)},$
 $(41, 17, 46, 33, 40, 25, 12, 45, 4, 15, 14, 7, 38, 23)_{\Theta(3,6,6)},$
 $(0, 6, 18, 47, 52, 49, 25, 12, 26, 15, 44, 38, 43, 3)_{\Theta(4,4,7)},$
 $(35, 43, 34, 45, 2, 41, 36, 21, 49, 32, 39, 44, 40, 22)_{\Theta(4,4,7)},$
 $(7, 44, 41, 23, 53, 3, 13, 29, 21, 17, 50, 16, 30, 35)_{\Theta(4,4,7)},$
 $(34, 17, 32, 21, 8, 22, 38, 2, 42, 10, 11, 20, 51, 41)_{\Theta(4,4,7)},$
 $(33, 16, 2, 12, 23, 27, 20, 35, 29, 52, 34, 48, 47, 44)_{\Theta(4,4,7)},$
 $(21, 19, 2, 41, 11, 22, 5, 48, 33, 16, 51, 0, 30, 49)_{\Theta(4,4,7)},$
 $(52, 27, 24, 14, 1, 22, 25, 35, 32, 30, 2, 44, 13, 49)_{\Theta(4,4,7)},$
 $(3, 9, 25, 48, 46, 38, 40, 43, 50, 33, 44, 1, 24, 31)_{\Theta(4,4,7)},$
 $(48, 11, 0, 16, 14, 1, 39, 38, 20, 54, 34, 50, 30, 43)_{\Theta(4,4,7)},$
 $(0, 36, 49, 38, 46, 35, 43, 19, 47, 34, 50, 48, 53, 23)_{\Theta(4,5,6)},$
 $(52, 36, 38, 3, 43, 50, 46, 32, 53, 4, 31, 7, 19, 15)_{\Theta(4,5,6)},$
 $(34, 26, 37, 47, 46, 49, 53, 17, 11, 12, 7, 38, 24, 25)_{\Theta(4,5,6)},$
 $(52, 18, 1, 53, 24, 10, 2, 39, 6, 35, 38, 11, 0, 5)_{\Theta(4,5,6)},$
 $(36, 50, 0, 9, 17, 6, 35, 42, 26, 38, 28, 37, 40, 13)_{\Theta(4,5,6)},$
 $(22, 21, 18, 47, 53, 33, 0, 14, 24, 37, 5, 48, 9, 44)_{\Theta(4,5,6)},$
 $(49, 51, 41, 7, 35, 6, 20, 5, 34, 3, 39, 38, 37, 1)_{\Theta(4,5,6)},$
 $(43, 40, 6, 0, 23, 21, 12, 41, 4, 9, 16, 14, 44, 31)_{\Theta(4,5,6)},$
 $(5, 27, 50, 39, 52, 17, 19, 14, 45, 30, 23, 7, 24, 47)_{\Theta(4,5,6)}$

under the action of the mapping $x \mapsto x + 5 \pmod{55}$.

K_{66} Let the vertex set be $Z_{65} \cup \{\infty\}$. The decompositions consist of the graphs

- $(\infty, 54, 56, 37, 27, 44, 1, 53, 18, 46, 14, 15, 49, 59)_{\Theta(1,2,12)},$
- $(54, 15, 58, 31, 30, 1, 39, 53, 7, 40, 35, 41, 57, 12)_{\Theta(1,2,12)},$
- $(31, 45, 0, 40, 21, 27, 36, 6, 23, 29, 3, 55, 64, 17)_{\Theta(1,2,12)},$
- $(60, 18, 47, 11, 43, 16, 57, 27, 19, 38, 45, 21, 26, 8)_{\Theta(1,2,12)},$
- $(48, 64, 28, 26, 33, 47, 16, 12, 37, 15, 43, 60, 10, 34)_{\Theta(1,2,12)},$
- $(9, 43, 42, 61, 64, 23, 8, 16, 17, 25, 15, 45, 57, 3)_{\Theta(1,2,12)},$
- $(53, 27, 22, 20, 38, 0, 14, 6, 46, 64, 10, 37, 51, 34)_{\Theta(1,2,12)},$
- $(55, 15, 62, 63, 0, 2, 58, 14, 26, 46, 43, 37, 22, 59)_{\Theta(1,2,12)},$
- $(43, 48, 27, 54, 35, 52, 14, 34, 22, 41, 15, 18, 16, 60)_{\Theta(1,2,12)},$
- $(22, 24, 64, 9, 5, 16, 44, 29, 0, 59, 58, 49, 56, 2)_{\Theta(1,2,12)},$
- $(8, 32, 61, 12, 56, 1, 5, 47, 21, 6, 48, \infty, 45, 29)_{\Theta(1,2,12)},$
- $(\infty, 29, 55, 25, 22, 23, 50, 51, 54, 26, 21, 17, 46, 27)_{\Theta(1,4,10)},$
- $(53, 47, 10, 63, 17, 19, 14, 58, 64, 18, 4, 28, 37, 30)_{\Theta(1,4,10)},$
- $(40, 43, 12, 24, 39, 50, 63, 28, 61, 10, 44, 5, 33, 53)_{\Theta(1,4,10)},$
- $(0, 36, 9, 8, 55, 21, 28, 17, 64, 45, 5, 63, 24, 40)_{\Theta(1,4,10)},$
- $(61, 8, 13, 56, 33, 54, 27, 53, 4, 17, 3, 5, 47, 19)_{\Theta(1,4,10)},$
- $(23, 18, 57, 35, 22, 2, 40, 55, 44, 36, 20, 53, 17, 56)_{\Theta(1,4,10)},$
- $(62, 46, 22, 1, 19, 52, 5, 36, 11, 12, 0, 8, 57, 33)_{\Theta(1,4,10)},$
- $(52, 4, 53, 51, 45, 2, 36, 12, 17, 37, 14, 59, 0, 29)_{\Theta(1,4,10)},$
- $(61, 50, 52, 9, 5, 43, 14, 15, 29, 6, 56, 11, 41, 33)_{\Theta(1,4,10)},$
- $(26, 9, 59, 46, 44, 35, 37, 45, 50, 17, 31, 3, 18, 21)_{\Theta(1,4,10)},$
- $(21, 11, 45, 6, 17, 64, 2, 34, 24, 33, 10, 54, 62, \infty)_{\Theta(1,4,10)},$
- $(\infty, 12, 8, 59, 11, 55, 54, 6, 37, 30, 63, 51, 2, 17)_{\Theta(1,6,8)},$
- $(52, 55, 17, 0, 10, 22, 59, 43, 57, 49, 13, 30, 9, 28)_{\Theta(1,6,8)},$
- $(3, 26, 36, 46, 44, 33, 51, 48, 55, 31, 7, 16, 10, 32)_{\Theta(1,6,8)},$
- $(9, 36, 54, 11, 30, 39, 57, 24, 10, 59, 33, 17, 38, 29)_{\Theta(1,6,8)},$
- $(18, 19, 59, 42, 3, 55, 25, 10, 5, 62, 63, 33, 36, 44)_{\Theta(1,6,8)},$
- $(35, 53, 55, 57, 10, 34, 1, 2, 6, 11, 47, 23, 28, 25)_{\Theta(1,6,8)},$
- $(7, 13, 20, 46, 63, 8, 51, 47, 28, 53, 22, 33, 49, 15)_{\Theta(1,6,8)},$
- $(40, 29, 56, 45, 5, 28, 59, 11, 31, 23, 21, 34, 46, 7)_{\Theta(1,6,8)},$
- $(1, 8, 15, 42, 9, 56, 14, 5, 61, 31, 3, 53, 0, 37)_{\Theta(1,6,8)},$
- $(30, 4, 31, 12, 64, 57, 2, 45, 11, 62, 58, 14, 19, 7)_{\Theta(1,6,8)},$
- $(15, 44, 58, 54, 51, 62, 17, 34, 24, 61, 11, 12, 35, \infty)_{\Theta(1,6,8)},$
- $(\infty, 45, 43, 34, 60, 18, 40, 37, 19, 58, 59, 47, 5, 39)_{\Theta(2,2,11)},$
- $(42, 14, 8, 52, 53, 33, 46, 48, 2, 30, 55, 0, 36, 7)_{\Theta(2,2,11)},$
- $(35, 43, 52, 47, 20, 2, 41, 31, 54, 9, 64, 14, 16, 1)_{\Theta(2,2,11)},$
- $(18, 10, 3, 58, 39, 64, 61, 62, 13, 0, 26, 40, 27, 24)_{\Theta(2,2,11)},$
- $(64, 53, 31, 35, 51, 27, 48, 42, 11, 36, 47, 40, 6, 18)_{\Theta(2,2,11)},$
- $(11, 24, 38, 29, 3, 39, 0, 6, 49, 47, 31, 13, 40, 45)_{\Theta(2,2,11)},$
- $(32, 21, 17, 7, 62, 54, 37, 5, 58, 53, 52, 38, 28, 20)_{\Theta(2,2,11)},$
- $(15, 27, 35, 14, 50, 7, 45, 8, 5, 38, 12, 48, 31, 4)_{\Theta(2,2,11)},$
- $(22, 1, 31, 54, 44, 60, 51, 59, 48, 29, 53, 4, 45, 61)_{\Theta(2,2,11)},$
- $(47, 28, 52, 19, 1, 8, 36, 29, 20, 22, 2, 61, 41, 24)_{\Theta(2,2,11)},$
- $(30, 38, 6, 41, 26, 5, 9, 39, 11, 32, \infty, 31, 50, 4)_{\Theta(2,2,11)},$

$(\infty, 15, 61, 23, 5, 0, 35, 22, 28, 32, 60, 54, 41, 21)_{\Theta(2,3,10)},$
 $(14, 16, 4, 10, 42, 55, 29, 59, 63, 22, 33, 9, 58, 41)_{\Theta(2,3,10)},$
 $(1, 17, 40, 36, 19, 29, 6, 7, 54, 59, 42, 20, 0, 63)_{\Theta(2,3,10)},$
 $(34, 6, 56, 12, 16, 13, 53, 33, 50, 2, 20, 48, 45, 24)_{\Theta(2,3,10)},$
 $(19, 62, 55, 28, 27, 8, 22, 38, 25, 9, 43, 31, 7, 59)_{\Theta(2,3,10)},$
 $(47, 38, 7, 41, 44, 59, 22, 30, 1, 15, 20, 58, 61, 5)_{\Theta(2,3,10)},$
 $(8, 44, 43, 34, 63, 51, 30, 19, 52, 55, 40, 36, 57, 2)_{\Theta(2,3,10)},$
 $(26, 46, 18, 12, 41, 42, 37, 8, 59, 60, 3, 47, 56, 45)_{\Theta(2,3,10)},$
 $(28, 9, 41, 13, 36, 50, 1, 48, 58, 31, 29, 2, 21, 55)_{\Theta(2,3,10)},$
 $(40, 32, 52, 2, 58, 49, 56, 15, 55, 43, 41, 48, 25, 39)_{\Theta(2,3,10)},$
 $(10, 27, 12, 44, 19, 3, 8, 41, 7, 61, 4, 24, 9, \infty)_{\Theta(2,3,10)},$
 $(\infty, 64, 24, 28, 7, 55, 21, 2, 51, 5, 60, 61, 41, 32)_{\Theta(2,4,9)},$
 $(56, 50, 8, 19, 30, 37, 42, 43, 31, 7, 52, 39, 62, 32)_{\Theta(2,4,9)},$
 $(34, 63, 24, 53, 4, 9, 1, 54, 33, 36, 57, 29, 5, 27)_{\Theta(2,4,9)},$
 $(52, 35, 56, 12, 63, 8, 21, 61, 46, 17, 15, 10, 22, 60)_{\Theta(2,4,9)},$
 $(40, 10, 1, 28, 43, 41, 17, 20, 26, 4, 5, 8, 32, 59)_{\Theta(2,4,9)},$
 $(3, 62, 57, 35, 21, 43, 5, 40, 32, 63, 33, 56, 46, 30)_{\Theta(2,4,9)},$
 $(36, 7, 63, 43, 38, 13, 47, 37, 59, 0, 19, 27, 53, 57)_{\Theta(2,4,9)},$
 $(45, 15, 63, 59, 36, 54, 23, 29, 32, 39, 4, 19, 64, 30)_{\Theta(2,4,9)},$
 $(39, 56, 57, 48, 19, 17, 36, 6, 58, 42, 5, 46, 41, 23)_{\Theta(2,4,9)},$
 $(43, 55, 19, 35, 39, 1, 30, 58, 54, 37, 31, 44, 46, 10)_{\Theta(2,4,9)},$
 $(13, 63, 64, 21, 25, 18, 41, 24, 12, \infty, 0, 44, 36, 29)_{\Theta(2,4,9)},$
 $(\infty, 44, 21, 22, 12, 53, 47, 5, 49, 17, 54, 59, 20, 61)_{\Theta(2,5,8)},$
 $(7, 45, 3, 41, 22, 17, 40, 34, 43, 48, 6, 62, 18, 32)_{\Theta(2,5,8)},$
 $(35, 44, 31, 23, 32, 62, 54, 4, 8, 29, 3, 38, 22, 24)_{\Theta(2,5,8)},$
 $(28, 25, 17, 47, 48, 58, 46, 13, 6, 42, 49, 51, 33, 34)_{\Theta(2,5,8)},$
 $(57, 18, 35, 7, 54, 23, 31, 51, 21, 61, 62, 37, 5, 34)_{\Theta(2,5,8)},$
 $(2, 24, 30, 60, 42, 54, 43, 44, 9, 41, 56, 25, 31, 5)_{\Theta(2,5,8)},$
 $(53, 36, 22, 39, 35, 45, 25, 16, 42, 21, 31, 51, 59, 41)_{\Theta(2,5,8)},$
 $(16, 15, 12, 32, 52, 25, 28, 54, 37, 61, 18, 58, 41, 4)_{\Theta(2,5,8)},$
 $(18, 60, 38, 51, 35, 21, 30, 16, 45, 46, 0, 28, 31, 9)_{\Theta(2,5,8)},$
 $(38, 60, 45, 44, 19, 4, 28, 2, 50, 52, 39, 46, 49, 13)_{\Theta(2,5,8)},$
 $(16, 20, 4, 27, 53, 45, 18, 43, \infty, 34, 10, 50, 62, 19)_{\Theta(2,5,8)},$
 $(\infty, 53, 44, 2, 26, 42, 3, 41, 5, 6, 31, 8, 62, 60)_{\Theta(2,6,7)},$
 $(10, 21, 28, 39, 24, 35, 58, 41, 44, 61, 62, 12, 55, 51)_{\Theta(2,6,7)},$
 $(19, 25, 10, 13, 57, 45, 17, 24, 2, 64, 1, 59, 6, 9)_{\Theta(2,6,7)},$
 $(63, 14, 34, 30, 60, 46, 5, 2, 33, 25, 36, 49, 17, 61)_{\Theta(2,6,7)},$
 $(8, 63, 2, 5, 62, 48, 20, 47, 53, 17, 0, 60, 35, 15)_{\Theta(2,6,7)},$
 $(34, 28, 62, 4, 59, 0, 38, 24, 20, 56, 43, 54, 33, 18)_{\Theta(2,6,7)},$
 $(25, 54, 12, 15, 21, 31, 0, 26, 16, 62, 55, 36, 22, 27)_{\Theta(2,6,7)},$
 $(42, 10, 31, 24, 64, 56, 47, 49, 6, 43, 48, 5, 58, 26)_{\Theta(2,6,7)},$
 $(24, 43, 46, 51, 8, 10, 42, 52, 32, 6, 21, 23, 4, 9)_{\Theta(2,6,7)},$
 $(54, 49, 30, 53, 14, 56, 22, 16, 38, 37, 13, 21, 26, 8)_{\Theta(2,6,7)},$
 $(5, 58, 18, 49, 62, 15, 57, 12, 9, 52, 17, 42, 46, \infty)_{\Theta(2,6,7)},$
 $(\infty, 59, 55, 35, 31, 6, 53, 62, 58, 27, 11, 61, 45, 2)_{\Theta(3,4,8)},$
 $(18, 20, 51, 23, 29, 34, 55, 5, 16, 19, 31, 50, 2, 21)_{\Theta(3,4,8)},$
 $(6, 62, 3, 26, 63, 38, 15, 12, 35, 28, 37, 17, 32, 27)_{\Theta(3,4,8)},$

$(40, 63, 45, 51, 6, 17, 10, 59, 44, 4, 3, 2, 38, 0)_{\Theta(3,4,8)},$
 $(40, 27, 16, 24, 18, 33, 13, 1, 62, 64, 36, 45, 55, 41)_{\Theta(3,4,8)},$
 $(52, 2, 4, 13, 62, 25, 29, 61, 26, 43, 15, 0, 48, 41)_{\Theta(3,4,8)},$
 $(27, 34, 53, 4, 14, 48, 13, 8, 46, 3, 60, 57, 32, 56)_{\Theta(3,4,8)},$
 $(57, 19, 55, 51, 63, 45, 37, 30, 17, 54, 3, 47, 35, 29)_{\Theta(3,4,8)},$
 $(12, 16, 46, 36, 45, 1, 64, 53, 29, 52, 19, 10, 49, 11)_{\Theta(3,4,8)},$
 $(20, 13, 56, 49, 9, 29, 45, 34, 38, 28, 12, 24, 6, 8)_{\Theta(3,4,8)},$
 $(6, 5, 29, 30, 27, 26, 39, 58, 19, 62, 4, 23, \infty, 34)_{\Theta(3,4,8)},$
 $(\infty, 52, 44, 43, 53, 59, 33, 50, 47, 17, 39, 32, 61, 26)_{\Theta(3,6,6)},$
 $(61, 48, 24, 53, 62, 17, 2, 43, 55, 10, 31, 52, 44, 42)_{\Theta(3,6,6)},$
 $(15, 21, 20, 48, 57, 29, 31, 19, 5, 4, 64, 55, 37, 2)_{\Theta(3,6,6)},$
 $(23, 0, 14, 40, 46, 42, 2, 64, 34, 45, 56, 1, 51, 4)_{\Theta(3,6,6)},$
 $(27, 37, 13, 48, 28, 44, 1, 5, 20, 14, 41, 33, 35, 8)_{\Theta(3,6,6)},$
 $(50, 21, 60, 7, 44, 63, 8, 52, 4, 53, 51, 17, 10, 46)_{\Theta(3,6,6)},$
 $(61, 47, 35, 36, 54, 34, 1, 9, 25, 13, 60, 27, 11, 14)_{\Theta(3,6,6)},$
 $(60, 26, 32, 13, 25, 4, 46, 52, 48, 41, 36, 56, 15, 23)_{\Theta(3,6,6)},$
 $(15, 23, 28, 3, 44, 59, 35, 26, 38, 48, 24, 12, 39, 57)_{\Theta(3,6,6)},$
 $(42, 5, 3, 36, 55, 35, 37, 14, 24, 1, 48, 4, 56, 28)_{\Theta(3,6,6)},$
 $(34, 5, 48, 32, 23, 19, 53, 46, \infty, 9, 10, 16, 7, 62)_{\Theta(3,6,6)},$
 $(\infty, 7, 30, 2, 51, 64, 47, 37, 42, 6, 43, 0, 50, 41)_{\Theta(4,4,7)},$
 $(31, 30, 0, 5, 12, 53, 59, 51, 57, 52, 10, 37, 44, 20)_{\Theta(4,4,7)},$
 $(33, 31, 45, 12, 18, 7, 52, 28, 10, 26, 9, 4, 2, 42)_{\Theta(4,4,7)},$
 $(15, 22, 26, 20, 4, 41, 53, 64, 37, 61, 23, 55, 2, 18)_{\Theta(4,4,7)},$
 $(56, 18, 60, 54, 8, 19, 39, 5, 52, 14, 53, 24, 25, 55)_{\Theta(4,4,7)},$
 $(42, 6, 64, 7, 8, 27, 63, 31, 53, 19, 10, 54, 4, 30)_{\Theta(4,4,7)},$
 $(39, 48, 21, 2, 23, 36, 27, 24, 6, 11, 49, 60, 20, 40)_{\Theta(4,4,7)},$
 $(47, 31, 59, 46, 24, 38, 56, 8, 61, 3, 48, 53, 39, 41)_{\Theta(4,4,7)},$
 $(24, 29, 37, 20, 38, 14, 18, 25, 60, 58, 1, 51, 5, 57)_{\Theta(4,4,7)},$
 $(43, 50, 57, 56, 36, 40, 42, 23, 58, 14, 63, 15, 29, 4)_{\Theta(4,4,7)},$
 $(18, 47, 48, 49, 14, \infty, 26, 55, 52, 46, 4, 16, 51, 50)_{\Theta(4,4,7)},$
 $(\infty, 40, 63, 31, 46, 45, 23, 17, 1, 16, 24, 37, 20, 58)_{\Theta(4,5,6)},$
 $(25, 7, 6, 46, 12, 29, 37, 15, 28, 17, 57, 27, 23, 34)_{\Theta(4,5,6)},$
 $(29, 8, 38, 57, 5, 34, 64, 36, 54, 58, 24, 22, 40, 11)_{\Theta(4,5,6)},$
 $(58, 52, 33, 9, 25, 11, 57, 3, 19, 46, 39, 59, 44, 34)_{\Theta(4,5,6)},$
 $(6, 54, 39, 2, 1, 48, 12, 16, 29, 49, 10, 58, 25, 55)_{\Theta(4,5,6)},$
 $(3, 6, 8, 51, 57, 18, 39, 5, 61, 16, 50, 27, 55, 11)_{\Theta(4,5,6)},$
 $(55, 4, 0, 41, 3, 64, 61, 34, 15, 60, 35, 42, 10, 25)_{\Theta(4,5,6)},$
 $(24, 41, 38, 15, 13, 28, 20, 21, 32, 12, 3, 17, 39, 25)_{\Theta(4,5,6)},$
 $(14, 46, 31, 2, 44, 17, 37, 61, 50, 7, 8, 28, 52, 4)_{\Theta(4,5,6)},$
 $(35, 10, 59, 0, 12, 28, 34, 8, 38, 32, 6, 36, 16, 30)_{\Theta(4,5,6)},$
 $(11, 37, 32, 47, 63, 3, 15, 44, \infty, 18, 8, 56, 58, 27)_{\Theta(4,5,6)}$

under the action of the mapping $x \mapsto x + 5 \pmod{65}$, $\infty \mapsto \infty$.

K_{70} Let the vertex set be $Z_{69} \cup \{\infty\}$. The decompositions consist of the graphs

$(\infty, 42, 47, 55, 9, 34, 6, 58, 22, 4, 31, 12, 2, 45)_{\Theta(1,2,12)},$
 $(17, 41, 44, 10, 53, 60, 32, 54, 20, 45, 3, 35, 64, 8)_{\Theta(1,2,12)},$
 $(56, 1, 39, 62, 48, 37, 34, 24, 10, 42, 3, 15, 26, 65)_{\Theta(1,2,12)},$

$(28, 48, 32, 24, 0, 33, 53, 16, 42, 55, 64, 10, 9, 17)_{\Theta(1,2,12)},$
 $(3, 10, 68, 2, 19, 29, 49, 25, 27, 50, 4, 39, 21, 15)_{\Theta(1,2,12)},$
 $(31, 1, 62, 32, 53, 55, 8, 21, 37, 15, 65, 50, 68, 49)_{\Theta(1,2,12)},$
 $(11, 23, 64, 9, 63, 15, 6, 35, 10, 16, 28, 57, 49, 14)_{\Theta(1,2,12)},$
 $(\infty, 45, 5, 48, 33, 28, 59, 10, 17, 61, 56, 13, 47, 43)_{\Theta(1,4,10)},$
 $(25, 23, 3, 53, 5, 43, 48, 38, 6, 49, 57, 39, 63, 17)_{\Theta(1,4,10)},$
 $(34, 51, 22, 19, 18, 23, 65, 35, 30, 38, 60, 63, 52, 12)_{\Theta(1,4,10)},$
 $(5, 3, 2, 26, 41, 19, 25, 67, 14, 39, 46, 55, 22, 44)_{\Theta(1,4,10)},$
 $(8, 37, 21, 58, 9, 24, 33, 49, 26, 43, 18, 64, 56, 65)_{\Theta(1,4,10)},$
 $(10, 24, 20, 54, 53, 66, 17, 5, 41, 40, 55, 23, 30, 3)_{\Theta(1,4,10)},$
 $(40, 9, 30, 26, 15, 36, 55, 5, 57, 2, 58, 19, 64, 43)_{\Theta(1,4,10)},$
 $(\infty, 40, 54, 21, 44, 49, 59, 2, 0, 11, 64, 3, 12, 60)_{\Theta(1,6,8)},$
 $(1, 68, 19, 4, 49, 61, 60, 67, 50, 47, 65, 44, 43, 29)_{\Theta(1,6,8)},$
 $(22, 64, 18, 30, 54, 38, 58, 33, 29, 36, 11, 53, 8, 23)_{\Theta(1,6,8)},$
 $(40, 65, 24, 55, 25, 39, 2, 15, 30, 33, 60, 32, 18, 52)_{\Theta(1,6,8)},$
 $(56, 39, 57, 26, 22, 58, 20, 67, 30, 37, 9, 4, 38, 29)_{\Theta(1,6,8)},$
 $(30, 52, 8, 42, 47, 14, 43, 48, 9, 19, 36, 56, 34, 26)_{\Theta(1,6,8)},$
 $(1, 24, 49, 56, 19, 21, 64, 47, 18, 5, 62, 36, 30, 43)_{\Theta(1,6,8)},$
 $(\infty, 47, 67, 65, 12, 59, 26, 30, 36, 5, 42, 4, 63, 1)_{\Theta(2,2,11)},$
 $(8, 42, 18, 63, 49, 59, 5, 29, 31, 37, 46, 38, 34, 54)_{\Theta(2,2,11)},$
 $(30, 29, 58, 34, 15, 55, 66, 68, 18, 19, 6, 48, 41, 10)_{\Theta(2,2,11)},$
 $(7, 31, 4, 43, 21, 65, 54, 53, 10, 27, 60, 9, 55, 63)_{\Theta(2,2,11)},$
 $(35, 20, 32, 14, 28, 12, 37, 38, 68, 52, 67, 56, 47, 34)_{\Theta(2,2,11)},$
 $(3, 46, 25, 12, 0, 64, 32, 67, 20, 62, 33, 68, 15, 2)_{\Theta(2,2,11)},$
 $(56, 38, 33, 30, 4, 28, 46, 3, 1, 31, 12, 29, 57, 18)_{\Theta(2,2,11)},$
 $(\infty, 5, 28, 60, 26, 59, 12, 45, 36, 38, 9, 31, 43, 46)_{\Theta(2,3,10)},$
 $(31, 22, 23, 12, 8, 24, 7, 63, 66, 56, 68, 50, 28, 51)_{\Theta(2,3,10)},$
 $(28, 45, 62, 2, 13, 49, 0, 5, 18, 50, 55, 46, 29, 19)_{\Theta(2,3,10)},$
 $(7, 38, 25, 45, 40, 18, 44, 24, 30, 34, 67, 28, 43, 27)_{\Theta(2,3,10)},$
 $(42, 56, 23, 24, 3, 52, 46, 19, 43, 14, 59, 10, 35, 41)_{\Theta(2,3,10)},$
 $(65, 4, 23, 56, 18, 51, 7, 6, 30, 64, 68, 2, 40, 12)_{\Theta(2,3,10)},$
 $(9, 65, 21, 32, 24, 63, 61, 8, 1, 38, 39, 0, 42, 35)_{\Theta(2,3,10)},$
 $(\infty, 19, 0, 65, 39, 60, 52, 14, 1, 56, 32, 63, 55, 31)_{\Theta(2,4,9)},$
 $(47, 20, 55, 10, 32, 59, 42, 0, 2, 68, 53, 34, 19, 57)_{\Theta(2,4,9)},$
 $(11, 60, 59, 57, 52, 66, 64, 22, 21, 54, 25, 8, 10, 5)_{\Theta(2,4,9)},$
 $(62, 54, 15, 55, 57, 45, 29, 4, 22, 65, 48, 56, 27, 3)_{\Theta(2,4,9)},$
 $(43, 40, 36, 4, 48, 61, 60, 28, 25, 51, 23, 35, 54, 20)_{\Theta(2,4,9)},$
 $(9, 57, 53, 55, 45, 42, 29, 58, 52, 3, 62, 51, 38, 10)_{\Theta(2,4,9)},$
 $(48, 47, 41, 32, 28, 38, 37, 3, 19, 8, 7, 16, 52, 29)_{\Theta(2,4,9)},$
 $(\infty, 62, 15, 50, 65, 37, 32, 1, 45, 54, 39, 4, 58, 28)_{\Theta(2,5,8)},$
 $(16, 49, 43, 42, 40, 21, 66, 48, 18, 8, 32, 53, 41, 67)_{\Theta(2,5,8)},$
 $(6, 21, 32, 3, 9, 27, 35, 44, 7, 45, 58, 49, 60, 20)_{\Theta(2,5,8)},$
 $(2, 38, 21, 35, 62, 40, 54, 55, 34, 18, 41, 32, 43, 3)_{\Theta(2,5,8)},$
 $(3, 46, 65, 23, 37, 34, 24, 64, 28, 33, 0, 42, 44, 50)_{\Theta(2,5,8)},$
 $(30, 10, 2, 51, 26, 1, 11, 34, 57, 37, 36, 23, 16, 62)_{\Theta(2,5,8)},$
 $(1, 18, 46, 50, 47, 65, 25, 13, 11, 15, 20, 7, 38, 6)_{\Theta(2,5,8)},$

$(\infty, 67, 10, 54, 1, 63, 37, 35, 59, 57, 39, 34, 14, 19)_{\Theta(2,6,7)},$
 $(22, 14, 40, 52, 67, 57, 17, 25, 63, 43, 10, 34, 7, 30)_{\Theta(2,6,7)},$
 $(26, 22, 19, 9, 17, 55, 21, 13, 7, 59, 41, 65, 18, 3)_{\Theta(2,6,7)},$
 $(2, 22, 35, 50, 0, 12, 47, 51, 55, 57, 48, 4, 38, 26)_{\Theta(2,6,7)},$
 $(18, 12, 57, 56, 2, 15, 21, 44, 38, 8, 14, 28, 66, 33)_{\Theta(2,6,7)},$
 $(2, 35, 30, 61, 57, 58, 11, 8, 1, 47, 3, 62, 34, 21)_{\Theta(2,6,7)},$
 $(58, 15, 52, 36, 63, 49, 66, 8, 0, 68, 28, 53, 44, 18)_{\Theta(2,6,7)},$
 $(\infty, 12, 28, 23, 17, 51, 54, 21, 41, 59, 19, 5, 48, 63)_{\Theta(3,4,8)},$
 $(2, 1, 43, 62, 26, 42, 3, 50, 51, 46, 60, 31, 9, 33)_{\Theta(3,4,8)},$
 $(6, 37, 64, 47, 29, 41, 50, 52, 43, 40, 1, 39, 27, 68)_{\Theta(3,4,8)},$
 $(15, 27, 65, 43, 8, 1, 5, 63, 3, 7, 61, 4, 53, 56)_{\Theta(3,4,8)},$
 $(19, 45, 35, 46, 37, 43, 64, 55, 21, 31, 32, 7, 44, 1)_{\Theta(3,4,8)},$
 $(2, 42, 32, 59, 39, 22, 56, 25, 51, 47, 11, 5, 7, 49)_{\Theta(3,4,8)},$
 $(5, 60, 66, 35, 3, 16, 40, 0, 6, 34, 42, 9, 65, 50)_{\Theta(3,4,8)},$
 $(\infty, 39, 63, 40, 53, 26, 48, 59, 7, 49, 21, 28, 62, 58)_{\Theta(3,6,6)},$
 $(35, 61, 63, 29, 7, 46, 4, 44, 28, 3, 17, 30, 42, 59)_{\Theta(3,6,6)},$
 $(37, 7, 49, 4, 29, 47, 11, 27, 63, 59, 9, 35, 30, 33)_{\Theta(3,6,6)},$
 $(15, 51, 4, 67, 19, 21, 60, 1, 44, 57, 43, 3, 9, 27)_{\Theta(3,6,6)},$
 $(14, 43, 37, 28, 64, 15, 24, 44, 50, 34, 3, 41, 62, 32)_{\Theta(3,6,6)},$
 $(58, 21, 66, 20, 36, 51, 47, 57, 65, 59, 2, 40, 50, 55)_{\Theta(3,6,6)},$
 $(48, 53, 27, 56, 43, 61, 5, 14, 28, 4, 25, 42, 44, 29)_{\Theta(3,6,6)},$
 $(\infty, 7, 57, 0, 48, 13, 58, 2, 8, 51, 35, 68, 53, 60)_{\Theta(4,4,7)},$
 $(37, 17, 62, 57, 39, 25, 30, 31, 54, 63, 48, 23, 21, 19)_{\Theta(4,4,7)},$
 $(19, 21, 35, 59, 17, 11, 34, 27, 63, 2, 37, 38, 6, 56)_{\Theta(4,4,7)},$
 $(59, 63, 31, 46, 24, 47, 57, 67, 50, 39, 11, 42, 15, 18)_{\Theta(4,4,7)},$
 $(32, 17, 64, 30, 43, 9, 40, 7, 26, 44, 23, 6, 55, 18)_{\Theta(4,4,7)},$
 $(45, 7, 12, 32, 28, 19, 2, 57, 16, 39, 25, 67, 5, 18)_{\Theta(4,4,7)},$
 $(20, 25, 40, 11, 14, 67, 61, 22, 1, 32, 2, 42, 34, 16)_{\Theta(4,4,7)},$
 $(\infty, 68, 7, 50, 48, 8, 65, 21, 29, 0, 24, 52, 46, 2)_{\Theta(4,5,6)},$
 $(32, 63, 14, 41, 15, 21, 68, 16, 6, 61, 48, 20, 1, 31)_{\Theta(4,5,6)},$
 $(39, 18, 56, 43, 10, 12, 3, 35, 37, 68, 62, 0, 14, 25)_{\Theta(4,5,6)},$
 $(14, 40, 4, 5, 28, 45, 60, 56, 20, 30, 55, 46, 67, 16)_{\Theta(4,5,6)},$
 $(62, 41, 2, 43, 65, 7, 23, 0, 46, 63, 52, 10, 17, 36)_{\Theta(4,5,6)},$
 $(53, 57, 38, 1, 18, 22, 6, 65, 61, 66, 0, 36, 67, 52)_{\Theta(4,5,6)},$
 $(0, 68, 1, 27, 33, 51, 22, 42, 7, 55, 57, 10, 13, 47)_{\Theta(4,5,6)}$

under the action of the mapping $x \mapsto x + 3 \pmod{69}$, $\infty \mapsto \infty$.

Proof of Lemma 3.12

$K_{15,15}$ Let the vertex set be Z_{30} partitioned according to residue class modulo 2. The decompositions consist of the graphs

$(0, 1, 3, 4, 5, 10, 17, 2, 11, 18, 7, 16, 27, 14)_{\Theta(1,3,11)},$
 $(0, 1, 3, 4, 9, 12, 7, 14, 5, 10, 21, 8, 25, 16)_{\Theta(1,5,9)},$
 $(0, 1, 3, 4, 9, 2, 11, 14, 13, 18, 29, 6, 25, 10)_{\Theta(1,7,7)},$
 $(0, 1, 3, 2, 5, 8, 7, 12, 21, 4, 17, 6, 25, 10)_{\Theta(3,3,9)},$
 $(0, 1, 3, 2, 5, 8, 15, 6, 11, 18, 7, 16, 29, 14)_{\Theta(3,5,7)}$

$(0, 1, 3, 2, 7, 8, 9, 12, 19, 6, 21, 10, 29, 14)_{\Theta(5,5,5)}$

under the action of the mapping $x \mapsto x + 2 \pmod{30}$.

$K_{15,20}$ Let the vertex set be $\{0, 1, \dots, 34\}$ partitioned into $\{0, 1, \dots, 19\}$ and $\{20, 21, \dots, 34\}$.

The decompositions consist of the graphs

$(0, 28, 30, 5, 22, 8, 27, 9, 29, 15, 21, 12, 23, 13)_{\Theta(1,3,11)},$
 $(11, 31, 28, 10, 27, 4, 29, 17, 21, 5, 33, 2, 20, 18)_{\Theta(1,3,11)},$
 $(9, 32, 22, 15, 26, 12, 28, 18, 29, 8, 31, 5, 25, 11)_{\Theta(1,3,11)},$
 $(32, 10, 7, 20, 16, 30, 2, 21, 19, 27, 6, 31, 3, 33)_{\Theta(1,3,11)},$
 $(0, 33, 23, 6, 21, 8, 20, 5, 29, 15, 30, 12, 31, 18)_{\Theta(1,5,9)},$
 $(22, 17, 7, 33, 1, 24, 8, 25, 0, 32, 13, 30, 19, 26)_{\Theta(1,5,9)},$
 $(28, 6, 3, 32, 15, 20, 0, 29, 13, 22, 5, 33, 14, 25)_{\Theta(1,5,9)},$
 $(33, 11, 15, 25, 13, 31, 6, 32, 10, 29, 4, 27, 2, 28)_{\Theta(1,5,9)},$
 $(0, 23, 26, 11, 28, 4, 25, 1, 24, 16, 29, 18, 32, 17)_{\Theta(1,7,7)},$
 $(20, 10, 7, 33, 9, 25, 18, 27, 14, 21, 1, 32, 15, 22)_{\Theta(1,7,7)},$
 $(22, 11, 4, 21, 0, 29, 7, 27, 9, 30, 1, 33, 2, 25)_{\Theta(1,7,7)},$
 $(1, 22, 29, 12, 25, 16, 27, 6, 28, 7, 21, 15, 23, 2)_{\Theta(1,7,7)},$
 $(0, 26, 21, 4, 29, 12, 24, 3, 23, 10, 32, 8, 33, 9)_{\Theta(3,3,9)},$
 $(3, 23, 20, 2, 31, 1, 27, 9, 30, 6, 28, 16, 22, 15)_{\Theta(3,3,9)},$
 $(13, 25, 24, 14, 20, 10, 31, 9, 23, 19, 33, 7, 22, 4)_{\Theta(3,3,9)},$
 $(6, 28, 25, 7, 21, 2, 27, 15, 26, 17, 31, 4, 33, 1)_{\Theta(3,3,9)},$
 $(0, 27, 29, 12, 22, 8, 21, 5, 24, 6, 30, 11, 33, 9)_{\Theta(3,5,7)},$
 $(29, 1, 3, 26, 8, 23, 7, 31, 2, 30, 18, 22, 4, 28)_{\Theta(3,5,7)},$
 $(33, 18, 3, 23, 14, 26, 9, 31, 16, 24, 13, 28, 15, 25)_{\Theta(3,5,7)},$
 $(7, 26, 25, 6, 33, 19, 22, 10, 20, 9, 29, 4, 31, 13)_{\Theta(3,5,7)},$
 $(0, 30, 25, 12, 33, 19, 28, 6, 26, 4, 21, 9, 24, 18)_{\Theta(5,5,5)},$
 $(11, 30, 23, 19, 32, 9, 22, 6, 27, 7, 21, 8, 28, 5)_{\Theta(5,5,5)},$
 $(23, 14, 15, 22, 19, 20, 6, 25, 18, 26, 12, 28, 13, 30)_{\Theta(5,5,5)},$
 $(32, 17, 0, 20, 1, 29, 8, 24, 14, 25, 13, 31, 15, 28)_{\Theta(5,5,5)}$

under the action of the mapping $x \mapsto x + 4 \pmod{20}$ for $x < 20$, $x \mapsto (x - 20 + 3 \pmod{15}) + 20$ for $x \geq 20$.

$K_{15,25}$ Let the vertex set be $\{0, 1, \dots, 39\}$ partitioned into $\{0, 1, \dots, 24\}$ and $\{25, 26, \dots, 39\}$.

The decompositions consist of the graphs

$(0, 34, 32, 12, 27, 17, 29, 5, 38, 18, 35, 22, 37, 9)_{\Theta(1,3,11)},$
 $(32, 2, 1, 37, 16, 36, 20, 30, 11, 27, 22, 26, 19, 34)_{\Theta(1,3,11)},$
 $(27, 9, 7, 30, 12, 37, 11, 26, 24, 25, 3, 35, 16, 38)_{\Theta(1,3,11)},$
 $(1, 30, 34, 15, 25, 20, 28, 10, 35, 8, 26, 4, 33, 3)_{\Theta(1,3,11)},$
 $(3, 27, 31, 8, 37, 4, 30, 18, 28, 1, 33, 24, 35, 20)_{\Theta(1,3,11)},$
 $(0, 36, 37, 18, 38, 14, 25, 17, 27, 15, 33, 3, 31, 1)_{\Theta(1,5,9)},$
 $(23, 38, 36, 2, 30, 3, 34, 5, 37, 6, 35, 19, 31, 4)_{\Theta(1,5,9)},$
 $(24, 31, 32, 9, 27, 16, 33, 6, 28, 12, 35, 2, 26, 5)_{\Theta(1,5,9)},$
 $(32, 17, 20, 29, 0, 26, 6, 30, 9, 28, 2, 25, 3, 36)_{\Theta(1,5,9)},$
 $(1, 26, 28, 4, 33, 13, 39, 15, 36, 22, 31, 23, 29, 11)_{\Theta(1,5,9)},$
 $(0, 34, 38, 19, 37, 9, 25, 1, 27, 11, 36, 4, 30, 7)_{\Theta(1,7,7)},$
 $(1, 28, 26, 23, 38, 10, 36, 2, 31, 0, 29, 5, 25, 8)_{\Theta(1,7,7)},$
 $(25, 12, 4, 26, 18, 33, 13, 36, 23, 30, 11, 29, 22, 35)_{\Theta(1,7,7)},$

$(23, 32, 35, 24, 36, 3, 34, 21, 28, 12, 30, 20, 33, 4)_{\Theta(1,7,7)},$
 $(19, 36, 25, 0, 39, 12, 32, 6, 29, 17, 34, 10, 26, 16)_{\Theta(1,7,7)},$
 $(0, 35, 32, 14, 33, 17, 31, 4, 30, 22, 29, 19, 28, 9)_{\Theta(3,3,9)},$
 $(29, 17, 11, 34, 3, 32, 13, 30, 16, 33, 10, 28, 12, 25)_{\Theta(3,3,9)},$
 $(16, 33, 36, 4, 31, 7, 28, 3, 37, 5, 27, 21, 32, 20)_{\Theta(3,3,9)},$
 $(8, 29, 33, 5, 37, 21, 36, 2, 35, 10, 34, 15, 32, 6)_{\Theta(3,3,9)},$
 $(14, 37, 33, 19, 28, 23, 29, 18, 36, 3, 26, 22, 39, 11)_{\Theta(3,3,9)},$
 $(0, 30, 31, 5, 25, 16, 27, 2, 29, 22, 35, 18, 32, 15)_{\Theta(3,5,7)},$
 $(24, 30, 37, 3, 36, 20, 27, 18, 31, 4, 32, 19, 38, 16)_{\Theta(3,5,7)},$
 $(18, 26, 38, 13, 33, 14, 27, 15, 28, 10, 25, 1, 35, 6)_{\Theta(3,5,7)},$
 $(22, 37, 28, 17, 38, 7, 27, 19, 26, 0, 35, 16, 36, 18)_{\Theta(3,5,7)},$
 $(16, 30, 31, 17, 28, 8, 34, 11, 39, 19, 35, 4, 37, 22)_{\Theta(3,5,7)},$
 $(0, 29, 27, 24, 26, 13, 37, 11, 31, 14, 30, 1, 28, 3)_{\Theta(5,5,5)},$
 $(6, 31, 27, 23, 34, 0, 30, 7, 37, 17, 38, 9, 36, 24)_{\Theta(5,5,5)},$
 $(18, 30, 26, 22, 29, 21, 35, 2, 36, 23, 25, 9, 34, 10)_{\Theta(5,5,5)},$
 $(12, 38, 30, 16, 28, 5, 31, 2, 26, 15, 36, 18, 34, 21)_{\Theta(5,5,5)},$
 $(29, 14, 24, 25, 13, 27, 11, 35, 20, 33, 12, 39, 10, 32)_{\Theta(5,5,5)}$

under the action of the mapping $x \mapsto x + 5 \pmod{25}$ for $x < 25$, $x \mapsto (x - 25 + 3 \pmod{15}) + 25$ for $x \geq 25$.

$K_{5,5,5}$ Let the vertex set be Z_{15} partitioned according to residue class modulo 3. The decompositions consist of the graphs

$(0, 1, 11, 2, 12, 10, 9, 14, 7, 5, 6, 4, 8, 3)_{\Theta(1,2,12)},$
 $(1, 2, 6, 5, 4, 9, 13, 8, 12, 14, 0, 10, 11, 7)_{\Theta(1,2,12)},$
 $(2, 3, 10, 13, 11, 12, 4, 14, 6, 8, 0, 7, 9, 5)_{\Theta(1,2,12)},$
 $(0, 13, 5, 4, 3, 7, 6, 11, 9, 8, 10, 14, 1, 12)_{\Theta(1,2,12)},$
 $(3, 13, 14, 11, 4, 2, 9, 1, 8, 7, 12, 5, 10, 6)_{\Theta(1,2,12)},$
 $(0, 1, 14, 13, 11, 5, 10, 6, 8, 4, 2, 9, 7, 12)_{\Theta(1,4,10)},$
 $(1, 2, 8, 12, 10, 5, 13, 6, 14, 9, 4, 11, 3, 7)_{\Theta(1,4,10)},$
 $(2, 3, 0, 10, 8, 12, 14, 1, 6, 5, 7, 11, 9, 13)_{\Theta(1,4,10)},$
 $(3, 4, 1, 9, 5, 10, 14, 7, 6, 11, 0, 8, 13, 12)_{\Theta(1,4,10)},$
 $(0, 4, 13, 2, 6, 7, 8, 9, 10, 11, 12, 5, 3, 14)_{\Theta(1,4,10)},$
 $(0, 1, 13, 11, 12, 2, 6, 8, 7, 5, 10, 9, 4, 14)_{\Theta(1,6,8)},$
 $(1, 2, 5, 12, 14, 9, 7, 11, 4, 6, 13, 3, 8, 10)_{\Theta(1,6,8)},$
 $(2, 3, 13, 14, 10, 6, 7, 9, 11, 0, 4, 8, 12, 1)_{\Theta(1,6,8)},$
 $(0, 5, 10, 12, 13, 8, 9, 2, 4, 3, 11, 7, 14, 6)_{\Theta(1,6,8)},$
 $(3, 5, 14, 0, 7, 12, 4, 10, 11, 6, 8, 1, 9, 13)_{\Theta(1,6,8)}$

as well as the graphs

$(0, 3, 1, 2, 4, 11, 7, 8, 10, 5, 9, 14, 6, 13)_{\Theta(2,2,11)},$
 $(0, 3, 1, 2, 7, 5, 4, 11, 12, 8, 10, 14, 6, 13)_{\Theta(2,3,10)},$
 $(0, 3, 1, 2, 4, 11, 10, 14, 9, 5, 6, 13, 8, 7)_{\Theta(2,4,9)},$
 $(0, 3, 1, 2, 4, 5, 10, 8, 12, 11, 7, 14, 9, 13)_{\Theta(2,5,8)},$
 $(0, 3, 1, 2, 4, 9, 14, 10, 11, 12, 5, 13, 8, 7)_{\Theta(2,6,7)},$
 $(0, 3, 1, 2, 4, 6, 13, 5, 10, 8, 12, 14, 7, 11)_{\Theta(3,4,8)},$

$$(0, 3, 1, 2, 4, 8, 6, 13, 11, 5, 10, 12, 7, 14)_{\Theta(3,6,6)},$$

$$(0, 3, 1, 2, 10, 4, 6, 14, 5, 7, 11, 9, 8, 13)_{\Theta(4,4,7)}$$

$$(0, 3, 1, 2, 7, 8, 12, 14, 10, 5, 6, 4, 11, 13)_{\Theta(4,5,6)}$$

under the action of the mapping $x \mapsto x + 3 \pmod{15}$

$K_{5,5,5,5}$ Let the vertex set be Z_{20} partitioned according to residue class modulo 4. The decompositions consist of the graphs

$$(0, 1, 2, 3, 4, 9, 6, 7, 5, 8, 14, 11, 16, 10)_{\Theta(1,2,12)},$$

$$(2, 7, 9, 4, 11, 0, 10, 5, 15, 1, 8, 17, 3, 14)_{\Theta(1,2,12)},$$

$$(0, 1, 2, 3, 4, 5, 6, 8, 11, 9, 7, 10, 19, 12)_{\Theta(1,4,10)},$$

$$(2, 7, 5, 10, 1, 8, 3, 13, 6, 0, 11, 17, 4, 14)_{\Theta(1,4,10)},$$

$$(0, 1, 2, 3, 4, 7, 10, 5, 6, 8, 14, 9, 11, 18)_{\Theta(1,6,8)},$$

$$(2, 7, 8, 1, 4, 11, 16, 9, 0, 10, 19, 5, 3, 17)_{\Theta(1,6,8)},$$

$$(0, 1, 2, 3, 5, 4, 10, 7, 6, 8, 15, 9, 12, 11)_{\Theta(2,2,11)},$$

$$(2, 3, 5, 8, 7, 13, 0, 9, 14, 4, 15, 6, 17, 10)_{\Theta(2,2,11)},$$

$$(0, 1, 2, 3, 4, 5, 7, 6, 8, 9, 14, 11, 13, 10)_{\Theta(2,3,10)},$$

$$(2, 3, 8, 7, 13, 9, 16, 5, 11, 4, 15, 6, 0, 10)_{\Theta(2,3,10)},$$

$$(0, 1, 2, 3, 4, 10, 5, 7, 6, 8, 9, 12, 19, 14)_{\Theta(2,4,9)},$$

$$(2, 3, 5, 8, 1, 6, 11, 16, 7, 13, 19, 9, 0, 10)_{\Theta(2,4,9)},$$

$$(0, 1, 2, 3, 4, 5, 7, 6, 8, 13, 10, 11, 14, 19)_{\Theta(2,5,8)},$$

$$(2, 3, 8, 9, 0, 7, 13, 15, 6, 1, 4, 14, 5, 12)_{\Theta(2,5,8)},$$

$$(0, 1, 2, 3, 4, 5, 7, 6, 9, 12, 10, 13, 8, 14)_{\Theta(2,6,7)},$$

$$(2, 3, 8, 7, 0, 10, 19, 6, 13, 11, 1, 15, 4, 17)_{\Theta(2,6,7)},$$

$$(0, 1, 2, 3, 5, 4, 7, 6, 8, 15, 10, 9, 12, 11)_{\Theta(3,4,8)},$$

$$(2, 3, 5, 10, 8, 1, 12, 9, 18, 7, 13, 11, 16, 6)_{\Theta(3,4,8)},$$

$$(0, 1, 2, 3, 5, 4, 7, 8, 6, 9, 10, 13, 11, 14)_{\Theta(3,6,6)},$$

$$(2, 3, 7, 12, 8, 1, 10, 4, 14, 15, 5, 19, 13, 16)_{\Theta(3,6,6)},$$

$$(0, 1, 2, 3, 4, 5, 6, 8, 7, 9, 11, 14, 17, 10)_{\Theta(4,4,7)},$$

$$(2, 3, 7, 1, 0, 8, 19, 9, 16, 11, 18, 13, 4, 14)_{\Theta(4,4,7)}$$

$$(0, 1, 2, 3, 4, 5, 6, 8, 11, 7, 9, 14, 17, 10)_{\Theta(4,5,6)},$$

$$(2, 3, 7, 1, 8, 11, 9, 0, 10, 19, 5, 4, 18, 12)_{\Theta(4,5,6)}$$

under the action of the mapping $x \mapsto x + 4 \pmod{20}$.